NCERT Solutions for Class 10

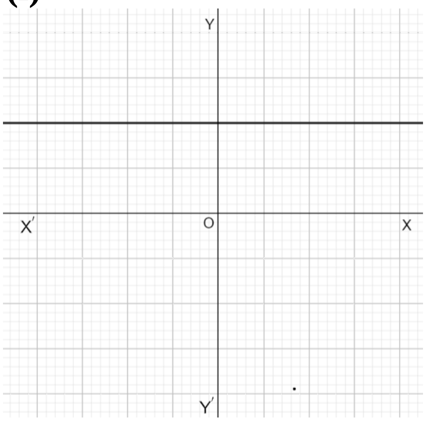
Mathematics

Chapter 2 – Polynomials

Exercise 2.1

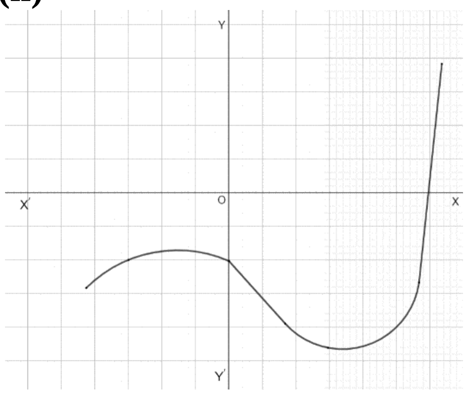
1. The graphs of $y=p(x)$ is given in following figure, for some Polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.

(i)



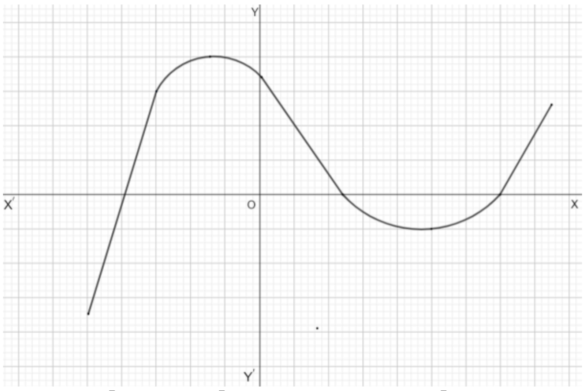
Ans: The graph does not intersect the x-axis at any point. Therefore, it does not have any zeroes.

(ii)



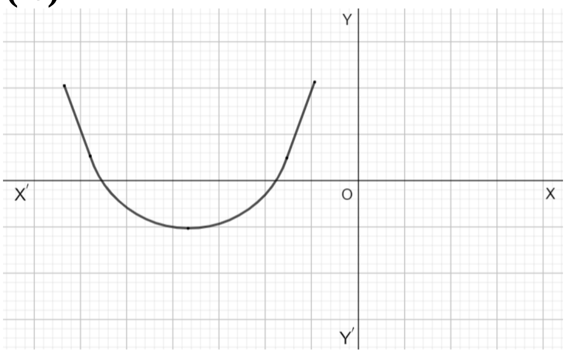
Ans: The graph intersects at the x-axis at only 1point. Therefore, the number of zeroes is 1.

(iii)



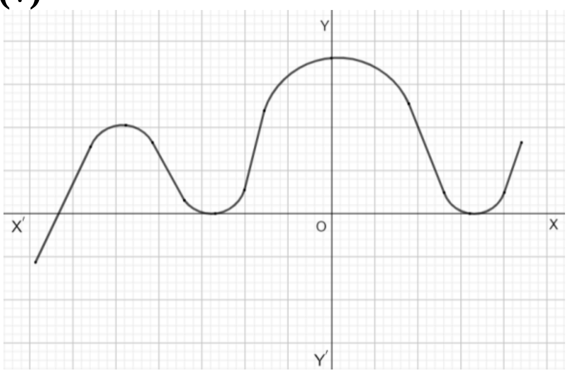
Ans: The graph intersects at the x-axis at 3 points. Therefore, the number of zeroes is 3.

(iv)



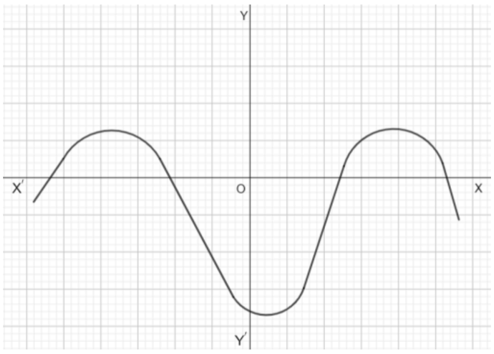
Ans: The graph intersects at the x-axis at 2 points. Therefore, the number of zeroes is 2.

(v)



Ans: The graph intersects at the x-axis at 4 points. Therefore, the number of zeroes is 4.

(vi)



Ans: The graph intersects at the x-axis at 3 points. Therefore, the number of zeroes is 3.

Exercise 2.2

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients. x^2-2x-8

Given: x^2-2x-8.

Now factorize the given polynomial to get the roots.

$\Rightarrow (x-4)(x+2)$

Ans: The value of x^2-2x-8 is zero.

when $x-4=0$ or $x+2=0$. i.e., $x=4$ or $x=-2$

Therefore, the zeroes of x^2-2x-8 are 4 and -2.

Now, Sum of zeroes $=4-2=2=-\frac{2}{1}=-\frac{\text{Coefficient of x}}{\text{Coefficient of x}^2}$

$\therefore \text{Sum of zeroes} = -\frac{\text{Coefficient of x}}{\text{Coefficient of x}^2}$

Product of zeroes $=4\times(-2)=-8=\frac{(-8)}{1}=\frac{\text{Constant term}}{\text{Coefficient of x}^2}$

$\therefore \text{Product of zeroes} = \frac{\text{Constant term}}{\text{Coefficient of x}^2}$.

(i) $4s^2-4s+1$

Ans: Given: $4s^2-4s+1$

Now factorize the given polynomial to get the roots.

$\Rightarrow (2s-1)^2$

The value of $4s^2-4s+1$ is zero.

when $2s-1=0$, $2s-1=0$. i.e., $s=\frac{1}{2}$ and $s=\frac{1}{2}$

Therefore, the zeroes of $4s^2 - 4s + 1$ are $\frac{1}{2}$ and $\frac{1}{2}$.

Now, Sum of zeroes $= \frac{1}{2} + \frac{1}{2} = 1 = \frac{(-4)}{4} = \frac{-(\text{Coefficient of s})}{\text{Coefficient of } s^2}$

$\therefore$ Sum of zeroes $= \frac{-(\text{Coefficient of s})}{\text{Coefficient of } s^2}$

Product of zeroes $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{Constant term}}{\text{Coefficient of } s^2}$

$\therefore$ Product of zeroes $= \frac{\text{Constant term}}{\text{Coefficient of } s^2}$.

(ii) $6x^2 - 3 - 7x$

Ans: Given: $6x^2 - 3 - 7x$

$\Rightarrow 6x^2 - 7x - 3$

Now factorize the given polynomial to get the roots.

$\Rightarrow (3x+1)(2x-3)$

The value of $6x^2 - 3 - 7x$ is zero.

when $3x+1=0$ or $2x-3=0$. i.e., $x = \frac{-1}{3}$ or $x = \frac{3}{2}$.

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$.

Now, Sum of zeroes $= \frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{Coefficient of x})}{\text{Coefficient of } x^2}$

$\therefore$ Sum of zeroes $= \frac{-(\text{Coefficient of x})}{\text{Coefficient of } x^2}$

Product of zeroes $= \frac{-1}{3} \times \frac{3}{2} = \frac{-3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$\therefore$ Product of zeroes $= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(iii) $4u^2 + 8u$

Ans: Given: $4u^2 + 8u$

$\Rightarrow 4u^2 + 8u + 0$

$\Rightarrow 4u(u+2)$

The value of $4u^2 + 8u$ is zero.

when $4u=0$ or $u+2=0$. i.e., $u = 0$ or $u = -2$

Therefore, the zeroes of $4u^2 + 8u$ are 0 and -2.

Now, Sum of zeroes $=0+(-2)=-2=\frac{-8}{4}=\frac{-(\text{Coefficient of u})}{\text{Coefficient of } u^2}$

$\therefore$ Sum of zeroes $=\frac{-(\text{Coefficient of u})}{\text{Coefficient of } u^2}$

Product of zeroes $=0\times(-2)=0=\frac{0}{4}=\frac{\text{Constant term}}{\text{Coefficient of } u^2}$

$\therefore$ Product of zeroes $=\frac{\text{Constant term}}{\text{Coefficient of } u^2}$

(iv) t^2-15

Ans: Given: t^2-15

$\Rightarrow t^2-0t-15$

Now factorize the given polynomial to get the roots.

$\Rightarrow (t-\sqrt{15})(t+\sqrt{15})$

The value of t^2-15 is zero.

when $t-\sqrt{15}=0$ or $t+\sqrt{15}=0$, i.e., $t=\sqrt{15}$ or $t=-\sqrt{15}$

Therefore, the zeroes of t^2-15 are $\sqrt{15}$ and $-\sqrt{15}$.

Now, Sum of zeroes $=\sqrt{15}+(-\sqrt{15})=0=\frac{-0}{1}=\frac{-(\text{Coefficient of t})}{\text{Coefficient of } t^2}$

$\therefore$ Sum of zeroes $=\frac{-(\text{Coefficient of t})}{\text{Coefficient of } t^2}$

Product of zeroes $=\left(\sqrt{15}\right)\times\left(-\sqrt{15}\right)=-15=\frac{-15}{1}=\frac{\text{Constant term}}{\text{Coefficient of } t^2}$

$\therefore$ Product of zeroes $=\frac{\text{Constant term}}{\text{Coefficient of } t^2}$.

(v) $3x^2-x-4$

Ans: Given: $3x^2-x-4$

Now factorize the given polynomial to get the roots.

$\Rightarrow \left(3x-4\right)(x+1)$

The value of $3x^2-x-4$ is zero.

when $3x-4=0$ or $x+1=0$, i.e., $x=\frac{4}{3}$ or $x=-1$

Therefore, the zeroes of $3x^2-x-4$ are $\frac{4}{3}$ and -1.

Now, Sum of zeroes $=\frac{4}{3}+(-1)=\frac{1}{3}=\frac{-(-1)}{3}=\frac{-(\text{Coefficient of x})}{\text{Coefficient of } x^2}$

$\therefore$ Sum of zeroes$=\frac{-(\text{Coefficient of x})}{\text{Coefficient of } x^2}$

Product of zeroes $=\frac{4}{3}\times(-1)=\frac{-4}{3}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$\therefore$ Product of zeroes $=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$.

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4},-1$

Ans:Given: $\frac{1}{4}$,-1

Let the zeroes of polynomial be α and β.

Then,

$\alpha+\beta=\frac{1}{4}$

$\alpha\beta=-1$

Hence, the required polynomial is $x^2-(\alpha+\beta)x+\alpha\beta$.

$\Rightarrow x^2-\frac{1}{4}x-1$

$\Rightarrow 4x^2-x-4$

Therefore, the quadratic polynomial is $4x^2-x-4$.

(ii) $\sqrt{2},\frac{1}{3}$

Ans: Given: $\sqrt{2},\frac{1}{3}$

Let the zeroes of polynomial be α and β.

Then,

$\alpha+\beta=\sqrt{2}$

$\alpha\beta=\frac{1}{3}$

Hence, the required polynomial is $x^2-(\alpha+\beta)x+\alpha\beta$.

$\Rightarrow x^2 - \sqrt{2}x + \frac{1}{3}$

$\Rightarrow 3x^2 - 3\sqrt{2}x+1$

Therefore, the quadratic polynomial is $3x^2 - 3\sqrt{2}x+1$.

(iii) $0, \sqrt{5}$

[here, root is missing]

Ans: Given: $0, \sqrt{5}$

Let the zeroes of polynomial be α and β.

Then,

$\alpha+\beta = 0$

$\alpha\beta = \sqrt{5}$

Hence, the required polynomial is $x^2 - (\alpha+\beta)x + \alpha\beta$.

$\Rightarrow x^2 - 0x + \sqrt{5}$

$\Rightarrow x^2 + \sqrt{5}$

Therefore, the quadratic polynomial is $x^2 + \sqrt{5}$.

(iv) 1,1

Ans: Given: 1,1

Let the zeroes of polynomial be α and β.

Then,

$\alpha+\beta = 1$

$\alpha\beta = 1$

Hence, the required polynomial is $x^2 - (\alpha+\beta)x + \alpha\beta$.

$\Rightarrow x^2 - 1x + 1$

Therefore, the quadratic polynomial is $x^2 - x+1$.

(v) $-\frac{1}{4}, \frac{1}{4}$

Ans: Given: $-\frac{1}{4}, \frac{1}{4}$

Let the zeroes of polynomial be α and β.

Then,

$\alpha+\beta = -\frac{1}{4}$

$\alpha\beta = \frac{1}{4}$

Hence, the required polynomial is $x^2 - (\alpha + \beta)x + \alpha\beta$.

$\Rightarrow x^2 - \left(-\frac{1}{4}\right)x + \frac{1}{4}$

$\Rightarrow 4x^2 + x + 1$

Therefore, the quadratic polynomial is $4x^2 + x + 1$**.**

(vi) 4,1

Ans: Given: 4,1

Let the zeroes of polynomial be α and β.

Then,

$\alpha + \beta = 4$

$\alpha\beta = 1$

Hence, the required polynomial is $x^2 - (\alpha + \beta)x + \alpha\beta$.

$\Rightarrow x^2 - 4x + 1$

Therefore, the quadratic polynomial is $x^2 - 4x + 1$.

Exercise 2.3

1. Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following:

(i) $p(x) = x^3 - 3x^2 + 5x - 3, g(x) = x^2 - 2$

Ans: Given: $p(x) = x^3 - 3x^2 + 5x - 3$ and $g(x) = x^2 - 2$

Then, divide the polynomial $p(x)$ by $g(x)$.

$$\begin{array}{r|l} & \quad x - 3 \\ \hline x^2 - 2 & x^3 - 3x^2 + 5x - 3 \\ & x^3 \qquad\quad - 2x \\ & \underline{- \qquad\quad +} \\ & \quad - 3x^2 + 7x - 3 \\ & \quad - 3x^2 \qquad + 6 \\ & \quad \underline{+ \qquad\quad -} \\ & \qquad\quad \underline{7x - 9} \end{array}$$

Therefore, Quotient $= x - 3$ and Remainder $= 7x - 9$.

(ii) $p(x)=x^4-3x^2+4x+5, g(x)=x^2+1-x$

Ans: Given: $p(x) = x^4-3x^2+4x+5 \Rightarrow x^4+0x^3-3x^2+4x+5$

$g(x) = x^2+1-x \Rightarrow x^2-x+1$

Then, divide the polynomial $p(x)$ by $g(x)$.

$$
\begin{array}{r|l}
 & x^2+x-3 \\
\hline
x^2-x+1 & x^4+0x^3-3x^2+4x+5 \\
 & x^4-x^3+x^2 \\
 & -\quad+\quad- \\
\hline
 & x^3-4x^2+4x+5 \\
 & x^3-x^2+x \\
 & -\quad+\quad- \\
\hline
 & -3x^2+3x+5 \\
 & -3x^2+3x-3 \\
 & +\quad-\quad+ \\
\hline
 & 8
\end{array}
$$

Therefore, Quotient $= x^2+x-3$ and Remainder $=8$.

(iii) $p(x)=x^4-5x+6, g(x)=2-x^2$

Ans: Given: $p(x) = x^4-5x+6 \Rightarrow x^4+0x^3+0x^2-5x+6$

$g(x) = 2-x^2 \Rightarrow -x^2+2$

Then, divide the polynomial $p(x)$ by $g(x)$.

$$
\begin{array}{r|l}
 & -x^2-2 \\
\hline
-x^2+2 & x^4+0x^3+0x^2-5x+6 \\
 & x^4\quad-2x^2 \\
 & -\quad+ \\
\hline
 & 2x^2-5x+6 \\
 & 2x^2\quad-4 \\
 & -\quad+ \\
\hline
 & -5x+10
\end{array}
$$

Therefore, Quotient $=-x^2-2$ and Remainder $=-5x+10$

2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

Ans: Given: $t^2 - 3,\ 2t^4 + 3t^3 - 2t^2 - 9t - 12$

Let us take first polynomial is $t^2 - 3 \Rightarrow t^2 + 0t - 3$.

And second polynomial is $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

Now, divide the second polynomial by first polynomial.

$$
\begin{array}{r|l}
 & \quad\quad\quad\quad 2t^2 + 3t + 4 \\
\cline{2-2}
t^2 + 0t^2 - 3 & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\
 & 2t^4 + 0t^3 - 6t^2 \\
 & \underline{-\quad -\quad +\quad\quad\quad\quad\quad} \\
 & \quad\quad 2t^3 + 4t^2 - 9t - 12 \\
 & \quad\quad 3t^3 + 0t^2 - 9t \\
 & \underline{\quad\quad -\quad -\quad +\quad\quad\quad} \\
 & \quad\quad\quad\quad 4t^2 + 0t - 12 \\
 & \underline{\quad\quad\quad\quad 4t^2 + 0t - 12} \\
 & \underline{\quad\quad\quad\quad\quad\quad 0 \quad\quad\quad}
\end{array}
$$

Since the remainder is 0

Therefore, $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

(ii) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$

Ans: Given: $x^2 + 3x + 1,\ 3x^4 + 5x^3 - 7x^2 + 2x + 2$

Let us take first polynomial is $x^2 + 3x + 1$.

And second polynomial is $3x^4 + 5x^3 - 7x^2 + 2x + 2$

Now, divide the second polynomial by first polynomial.

$$
\begin{array}{r|l}
 & 3x^2 - 4x + 2 \\
\hline
x^2 + 3x + 1 & 3x^4 + 5x^3 - 7x^2 + 2x + 2 \\
 & 3x^4 + 9x^3 + 3x^2 \\
 & - \quad - \quad - \\
\hline
 & -4x^3 - 10x^2 + 2x + 2 \\
 & -4x^3 - 12x^2 - 4x \\
 & + \quad + \quad + \\
\hline
 & 2x^2 + 6x + 2 \\
 & 2x^2 + 6x + 2 \\
 & - \quad - \quad - \\
\hline
 & 0 \\
\hline
\end{array}
$$

Since the remainder is 0

Therefore, $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

(iii) $x^2 - 3x + 1$, $x^5 - 4x^3 + x^2 + 3x + 1$

Ans: Given: $x^2 - 3x + 1$, $x^5 - 4x^3 + x^2 + 3x+1$.

Let us take first polynomial is $x^2 - 3x + 1$.

And second polynomial is $x^5 - 4x^3 + x^2 + 3x+1$.

Now, divide the second polynomial by first polynomial.

$$
\begin{array}{r|l}
 & x^2 - 1 \\
\hline
x^2 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x + 1 \\
 & x^5 - 3x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & -x^3 \qquad + 3x + 1 \\
 & -x^3 \qquad + 3x - 1 \\
 & + \qquad - \quad + \\
\hline
 & 2 \\
\hline
\end{array}
$$

Since the remainder $\neq 0$.

Therefore, $x^2 - 3x + 1$ is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$.

3. Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

Ans: Let us assume, $p(x)=3x^4+6x^3-2x^2-10x-5$

Then, given two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

$\therefore \left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)=\left(x^2-\frac{5}{3}\right)$ is a factor of $3x^4+6x^3-2x^2-10x-5$

Now, divide the given polynomial by $x^2-\frac{5}{3}$

$$
\begin{array}{r|l}
 & 3x^2+6x+3 \\
x^2+0x-\frac{5}{3} & 3x^4+6x^3-2x^2-10x-5 \\
 & 3x^4+0x^3-5x^2 \\
 & -\quad -\quad + \\ \hline
 & 6x^3+3x^2-10x-5 \\
 & 6x^3+0x^2-10x \\
 & -\quad -\quad + \\ \hline
 & 3x^2+0x-5 \\
 & 3x^2+0x-5 \\
 & -\quad -\quad + \\ \hline
 & 0 \\ \hline
\end{array}
$$

$\therefore 3x^4+6x^3-2x^2-10x-5=\left(x^2-\frac{5}{3}\right)\left(3x^2+6x+3\right)$

$\Rightarrow 3\left(x^2-\frac{5}{3}\right)\left(x^2+2x+1\right)$

$\Rightarrow \left(3x^2-5\right)\left(x^2+2x+1\right)$

Now, factorize the polynomial x^2+2x+1

$\Rightarrow (x+1)^2$

Hence, its zero is given by $x+1=0$

$\Rightarrow x = -1$

As it has the term $(x+1)^2$, then, there will be 2 zeroes at $x = -1$.

Therefore, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -1$ and -1.

4. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find g(x).

Ans: Let us take the dividend as $p(x)$. Then, $p(x) = x^3 - 3x^2 + x + 2$

And the divisor is $g(x)$. Then, find the value of $g(x)$.

Quotient $= (x - 2)$

Remainder $= (-2x + 4)$

Dividend = Divisor × Quotient + Remainder

$x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) + (-2x + 4)$

$x^3 - 3x^2 + x + 2 + 2x - 4 = g(x) \times (x - 2)$

$x^3 - 3x^2 + 3x - 2 = g(x) \times (x - 2)$

Hence, $g(x)$ is the quotient when we divide $(x^3 - 3x^2 + 3x - 2)$ by $(x - 2)$.

$$\begin{array}{r|l} & x^2 - x + 1 \\ \hline x - 2 & x^3 - 3x^2 + 3x - 2 \\ & x^3 - 2x^2 \\ & - \quad + \\ \hline & -x^2 + 3x - 2 \\ & -x^2 + 2x \\ & + \quad - \\ \hline & x - 2 \\ & x - 2 \\ & - \quad + \\ \hline & 0 \\ \hline \end{array}$$

$\therefore g(x) = (x^2 - x + 1)$

5. Give examples of polynomial p(x), g(x), q(x) and r(x), which satisfy the division algorithm and

(i) $\deg p(x) = \deg q(x)$

Ans: According to the division algorithm, if p(x) and g(x) are two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that

$p(x) = g(x) \times q(x) + r(x)$,

Where $r(x)=0$ or degree of $r(x) <$ degree of $g(x)$

Degree of a polynomial is the highest power of the variable in the polynomial.

Given: $\deg p(x) = \deg q(x)$

Degree of quotient will be equal to degree of dividend when divisor is constant (i.e., when any polynomial is divided by a constant).

Let us assume the division of $6x^2 + 2x + 2$ by 2.

Here, $p(x) = 6x^2 + 2x + 2$

$g(x) = 2$

$q(x) = 3x^2 + x + 1$ and $r(x) = 0$

Degree of $p(x)$ and $q(x)$ is the same i.e., 2.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$6x^2 + 2x + 2 = 2\left(3x^2 + x + 1\right)$

$6x^2 + 2x + 2 = 6x^2 + 2x + 2$

Therefore, the division algorithm is satisfied.

(ii) $\deg q(x) = \deg r(x)$

Ans: Given: $\deg q(x) = \deg r(x)$

Let us assume the division of $x^3 + x$ by x^2

Here, $p(x) = x^3 + x$

$g(x) = x^2$

$q(x) = x$ and $r(x) = x$

Clearly, the degree of q(x) and r(x) is the same, i.e.,

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3 + x = \left(x^2\right) \times x + x$

$x^3 + x = x^3 + x$

Therefore, the division algorithm is satisfied.

(iii) $\deg r\left(x\right) = 0$

Ans: Given: $\deg r(x) = 0$

Degree of remainder will be 0 when remainder comes to a constant.

Let us assume the division of x^3+1 by x^2.

Here, $p(x) = x^3+1$

$g(x) = x^2$

$q(x) = x$ and $r(x) = 1$

Clearly, the degree of $r(x)$ is 0

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3+1 = \left(x^2\right) \times x + 1$

$x^3+1 = x^3+1$

Therefore, the division algorithm is satisfied.

Exercise 2.4

1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3 + x^2 - 5x + 2;\ \frac{1}{2}, 1, -2$

Ans: Let us assume $p(x) = 2x^3 + x^2 - 5x + 2$

And zeroes for this polynomial are $\frac{1}{2}, 1, -2$. Then,

$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$

$= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2$

$= 0$

$p(1) = 2\times1^3 + 1^2 - 5\times1 + 2$

$= 0$

$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$

$= -16 + 4 + 10 + 2$

$= 0$

Therefore, $\frac{1}{2}, 1$ and -2 are the zeroes of the given polynomial.

Comparing the given polynomial with ax^3+bx^2+cx+d to get, $a = 2, b = 1, c = -5,$ and $d = 2$.

Let us take $\alpha = \frac{1}{2}, \beta = 1,$ and $\gamma = -2$.

Then,

$$\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = -\frac{1}{2} = \frac{-b}{a}$$

$$\therefore \alpha + \beta + \gamma = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{1}{2} \times 1 + 1(-2) + \frac{1}{2}(-2) = \frac{-5}{2} = \frac{c}{a}$$

$$\therefore \alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = \frac{-1}{1} = \frac{-(2)}{2} = \frac{-d}{a}$$

$$\therefore \alpha\beta\gamma = \frac{-d}{a}$$

Therefore, the relationship between the zeroes and the coefficients is verified.

(ii) $x^3 - 4x^2 + 5x - 2$; 2,1,1

Ans: Let us assume $p(x) = x^3 - 4x^2 + 5x - 2$

And zeroes for this polynomial are $2,1,1$. Then,

$$p(2) = 2^3 - 4(2^2) + 5(2) - 2$$
$$= 8 - 16 + 10 - 2$$
$$= 0$$

$$p(1) = 1^3 - 4(1^2) + 5(1) - 2$$
$$= 1 - 4 + 5 - 2$$
$$= 0$$

Therefore, $2, 1,$ and 1 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$ to get, $a = 1$, $b = -4$, $c = 5$, and $d = -2$.

Let us take $\alpha = 2, \beta = 1,$ and $\gamma = 1$.

Then,

$$\alpha + \beta + \gamma = 2 + 1 + 1 = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$$

$$\therefore \alpha + \beta + \gamma = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = (2)(1) + (1)(1) + (2)(1)$$

$= 2+1+2 = 5 = \frac{(5)}{1} = \frac{c}{a}$

$\therefore \alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$

$\alpha\beta\gamma = 2\times1\times1 = 2 = \frac{-(-2)}{1} = \frac{-d}{a}$

$\therefore \alpha\beta\gamma = \frac{-d}{a}$

Therefore, the relationship between the zeroes and the coefficients is verified.

2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as $2, -7, -14$ respectively.

Ans: Let the polynomial be $ax^3 + bx^2 + cx + d$ and the zeroes be $\alpha, \beta,$ and γ.

Then given that,

$\alpha+\beta+\gamma = \frac{2}{1} = \frac{-b}{a}$

$\alpha\beta+\beta\gamma+\alpha\gamma = \frac{-7}{1} = \frac{c}{a}$

$\alpha\beta\gamma = \frac{-14}{1} = \frac{-d}{a}$

If $a=1$, then $b = -2, c = -7, d = 14$

Therefore, the polynomial is $x^3 - 2x^2 - 7x + 14$.

3. If the zeroes of polynomial $x^3 - 3x^2 + x + 1$ are $a - b, a, a+b$, find a and b.

Ans: Let us assume $p(x) = x^3 - 3x^2 + x + 1$.

And the zeroes are $a-b, a, a+b$.

Let us assume $\alpha = a-b, \beta = a$ and $\gamma = a+b$.

Comparing the given polynomial with $px^3 + qx^2 + rx + t$ to get,

$p = 1, q = -3, r = 1,$ and $t = 1$.

Then,

$\alpha+\beta+\gamma = a - b + a + a+b$

$\Rightarrow \frac{-q}{p} = 3a$

$\Rightarrow \frac{-(-3)}{1} = 3a$

$\Rightarrow 3 = 3a$

$\therefore a = 1$

Then, the zeroes are $1-b, 1+b$.

Now,

$\alpha\beta\gamma = 1(1-b)(1+b)$

$\Rightarrow \dfrac{-t}{p} = 1 - b^2$

$\Rightarrow \dfrac{-1}{1} = 1 - b^2$

$\Rightarrow 1 - b^2 = -1$

$\Rightarrow b^2 = 2$

$\therefore b = \pm\sqrt{2}$

Therefore, $a=1$ and $b = \pm\sqrt{2}$.

4. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Ans: Given that $2+\sqrt{3}$ and $2-\sqrt{3}$ are zeroes of the given polynomial.

Therefore, $\left(x - 2 - \sqrt{3}\right)\left(x - 2 + \sqrt{3}\right) = x^2 + 4 - 4x - 3$

$= x^2 - 4x + 1$

Hence, $x^2 - 4x + 1$ is a factor of the given polynomial.

For finding the remaining zeroes of the given polynomial, we will find the quotient by dividing the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ by $x^2 - 4x + 1$.

$$
\begin{array}{r|l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 + x^2 \\
 & - \quad + \quad - \\
\cline{2-2}
 & -2x^3 - 27x^2 + 138x - 35 \\
 & -2x^3 + 8x^2 - 2x \\
 & + \quad - \quad + \\
\cline{2-2}
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & + \quad - \quad + \\
\cline{2-2}
 & 0 \\
\cline{2-2}
\end{array}
$$

Clearly, $x^4 - 6x^3 - 26x^2 + 138x - 35 = \left(x^2 - 4x + 1\right)\left(x^2 - 2x - 35\right)$

Then, $x^2-2x-35$ is also a factor of the given polynomial.

And, $x^2-2x-35=(x-7)(x+5)$

Therefore, the value of the polynomial is also zero when $x-7=0$ Or $x+5=0$

Hence, $x=7$ or -5

Therefore, 7 and -5 are also zeroes of this polynomial.

5. If the polynomial $x^4-6x^3+16x^2-25x-10$ is divided by another polynomial x^2-2x+k, the remainder comes out to be $x+a$, find k and a.

Ans: Given: $x^4-6x^3+16x^2-25x-10$ and x^2-2x+k.

Then, the remainder is $x+a$

By division algorithm,

Dividend = Divisor×Quotient+Remainder

Dividend-Remainder= Divisor×Quotient

$x^4-6x^3+16x^2-25x-10-x-a \Rightarrow x^4-6x^3+16x^2-26x+10-a$ will be perfectly divisible by x^2-2x+k.

Let us divide $x^4-6x^3+16x^2-26x+10-a$ by x^2-2x+k

$$
\begin{array}{r|l}
 & x^2-4x+(8-k) \\
\hline
x^2-2x+k & x^4-6x^3+16x^2-26x+10-a \\
 & x^4-2x^3+kx^2 \\
 & -\ \ +\ \ - \\
\hline
 & -4x^3+(16-k)x^2-26x \\
 & -4x^3+\ 8x^2-4kx \\
 & +\ \ -\ \ + \\
\hline
 & (8-k)x^2-(26-4k)x+10-a \\
 & (8-k)x^2-(16-2k)x+(8k-k^2) \\
 & -\ \ +\ \ - \\
\hline
 & (-10+2k)x+(10-a-8k+k^2) \\
\hline
\end{array}
$$

Hence, the reminder $(-10+2k)x+(10-a-8k+k^2)$ will be 0.

Then, $(-10+2k)=0$ and $(10-a-8k+k^2)=0$

For $(-10+2k)=0$

$2k=10$

$\therefore k=5$

For $\left(10-a-8k+k^2\right)=0$

$10-a-8\times5+25=0$

$10-a-40+25=0$

$-5-a=0$

$\therefore a=-5$

Hence, $k=5$ and $a=-5$.