Maths Class 10 NCERT Solutions

Chapter 7 – Coordinate Geometry

Exercise 7.1

1. Find the distance between the following pairs of points:

(i) $(2,3),(4,1)$

Ans: Given that,

Let the points be $(2,3)$ and $(4,1)$

To find the distance between the points $(2,3),(4,1)$.

Distance between two points is given by the Distance formula

$=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

Here, $x_1=2$

$x_2=4$

$y_1=3$

$y_2=1$

Thus, the distance between $(2,3)$ and $(4,1)$ is given by,

$d=\sqrt{(2-4)^2+(3-1)^2}$

$=\sqrt{(-2)^2+(2)^2}$

$=\sqrt{4+4}$

$=\sqrt{8}$

$=2\sqrt{2}$

$\therefore$ The distance between $(2,3)$ and $(4,1)$ is $2\sqrt{2}$ units.

(ii) $(-5,7),(-1,3)$

Ans: Given that,

Let the points be $(-5,7)$ and $(-1,3)$

To find the distance between the points $(-5,7),(-1,3)$.

Distance between two points is given by the Distance formula

$=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

Here, $x_1=-5$

$x_2=-1$

$y_1=7$

$y_2=3$

Thus, the distance between $(-5,7)$ and $(-1,3)$ is given by,

$d=\sqrt{(-5-(-1))^2+(7-3)^2}$

$=\sqrt{(-4)^2+(4)^2}$

$=\sqrt{16+16}$

$=\sqrt{32}$

$=4\sqrt{2}$

$\therefore$ The distance between $(-5,7)$ and $(-1,3)$ is $4\sqrt{2}$ units.

(iii) $(a,b),(-a,-b)$

Ans: Given that,

Let the points be (a,b) and $(-a,-b)$

To find the distance between the points $(a,b),(-a,-b)$.

Distance between two points is given by the Distance formula

$=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

Here, $x_1=a$

$x_2=-a$

$y_1=b$

$y_2=-b$

Thus, the distance between (a,b) and $(-a,-b)$ is given by,

$$d=\sqrt{(a-(-a))^2+(b-(-b))^2}$$
$$=\sqrt{(2a)^2+(2b)^2}$$
$$=\sqrt{4a^2+4b^2}$$
$$=\sqrt{4}\sqrt{a^2+b^2}$$
$$=2\sqrt{a^2+b^2}$$

$\therefore$ The distance between (a,b) and $(-a,-b)$ is $2\sqrt{a^2+b^2}$ units.

2. Find the distance between the points $(0,0)$ and $(36,15)$. Can you now find the distance between the two towns A and B discussed in Section 7.2?

Ans: Given that,

Let the points be $(0,0)$ and $(36,15)$

To find the distance between the points $(0,0),(36,15)$.

Distance between two points is given by the Distance formula

$$=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

Here, $x_1=0$

$x_2=36$

$y_1=0$

$y_2=15$

Thus, the distance between $(0,0)$ and $(36,15)$ is given by,

$$d=\sqrt{(0-36)^2+(0-15)^2}$$
$$=\sqrt{(-36)^2+(-15)^2}$$
$$=\sqrt{1296+225}$$
$$=\sqrt{1521}$$
$$=39$$

Yes, it is possible to find the distance between the given towns A and B. The positions of this town are $A(0,0)$ and $B(36,15)$. And it can be calculated as above.

$\therefore$ The distance between $A(0,0)$ and $B(36,15)$ is 39 km.

3. Determine if the points $(1,5),(2,3)$ and $(-2,-11)$ are collinear.

Ans: Given that,

Let the three points be $(1,5),(2,3)$ and $(-2,-11)$

To determine if the given points are collinear

Let $A(1,5), B(2,3), C(-2,-11)$ be the vertices of the given triangle.

The distance between any two points is given by the Distance formula,

$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

To find the distance between the points $A(1,5)$ and $B(2,3)$

$x_1=1$

$x_2=2$

$y_1=5$

$y_2=3$

$AB=\sqrt{(1-2)^2+(5-3)^2}$

$=\sqrt{(-1)^2+(2)^2}$

$=\sqrt{1+4}$

$=\sqrt{5}$

To find the distance between the points $B(2,3)$ and $C(-2,-11)$

$x_1=2$

$x_2=-2$

$y_1=3$

$y_2=-11$

$BC=\sqrt{\left(2-(-2)\right)^2+\left(3-(-11)\right)^2}$

$=\sqrt{(4)^2+(14)^2}$

$=\sqrt{16+196}$

$=\sqrt{212}$

To find the distance between the points $A(1,5)$ and $C(-2,-11)$

$x_1=1$

$x_2=-2$

$y_1=5$

$y_2=-11$

$CA=\sqrt{\left(1-(-2)\right)^2+\left(5-(-11)\right)^2}$

$=\sqrt{(3)^2+(16)^2}$

$=\sqrt{9+256}$

$=\sqrt{265}$

Since $AB+AC\neq BC$ and $AB\neq BC+AC$
and $AC\neq BC$

$\therefore$ The points $A(1,5), B(2,3), C(-2,-11)$ are not collinear.

4. Check whether $(5,-2),(6,4)$ and $(7,-2)$ are the vertices of an isosceles triangle.

Ans: Given that,

Let the three points be $(5,-2),(6,4)$ and $(7,-2)$ are the vertices of the triangle.

To determine if the given points are the vertices of an isosceles triangle.

Let $A(5,-2), B(6,4), C(7,-2)$ be the vertices of the given triangle.

The distance between any two points is given by the Distance formula,

$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

To find the distance between the points $A(5,-2)$ and $B(6,4)$

$x_1 = 5$

$x_2 = 6$

$y_1 = -2$

$y_2 = 4$

$AB = \sqrt{(5-6)^2 + (-2-4)^2}$

$= \sqrt{(-1)^2 + (-6)^2}$

$= \sqrt{1+36}$

$= \sqrt{37}$

To find the distance between the points $B(6,4)$ and $C(7,-2)$

$x_1 = 6$

$x_2 = 7$

$y_1 = 4$

$y_2 = -2$

$BC = \sqrt{(6-7)^2 + (4-(-2))^2}$

$= \sqrt{(-1)^2 + (6)^2}$

$= \sqrt{1+36}$

$= \sqrt{37}$

To find the distance between the points $A(5,-2)$ and $C(7,-2)$

$x_1 = 5$

$x_2 = 7$

$y_1 = -2$

$y_2 = -2$

$CA = \sqrt{(5-7)^2 + (-2-(-2))^2}$

$= \sqrt{(-2)^2 + (0)^2}$

$= \sqrt{4+0}$

$= 2$

We can conclude that $AB = BC$.

Since two sides of the triangle are equal in length, ABC is an isosceles triangle.

5. In a classroom, 4 friends are seated at the points A, B, C and D are shown in the following figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees.

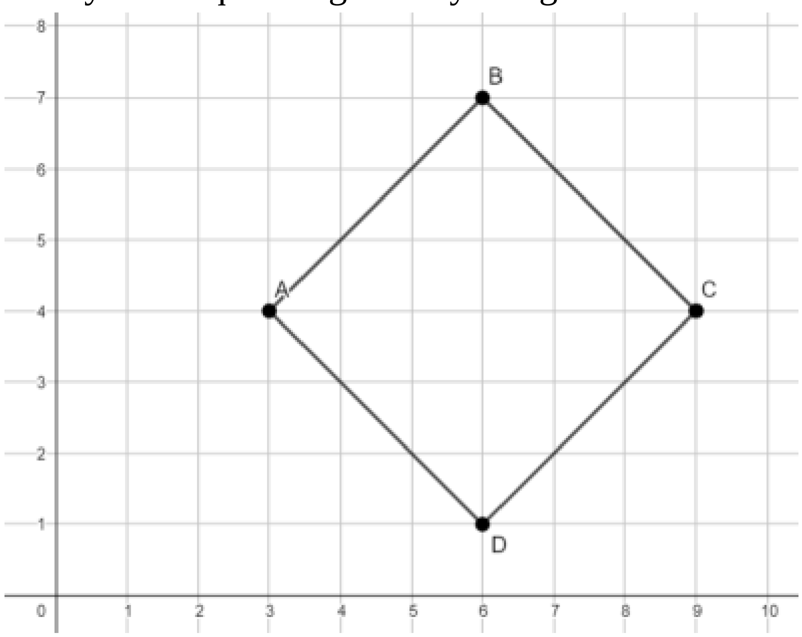
Using the distance formula, find which of them is correct.

Ans: Given that,

4 friends are seated at the points A, B, C, D

To find,

If they form square together by using distance formula



From the figure, we observe the points $A(3, 4), B(6, 7), C(9, 4)$ and $D(6, 1)$ are the positions of the four students.

The distance between any two points is given by the Distance formula,

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

To find the distance between the points $A(3, 4)$ and $B(6, 7)$

$x_1 = 3$

$x_2 = 6$

$y_1 = 4$

$y_2 = 7$

$AB = \sqrt{(3-6)^2 + (4-7)^2}$

$= \sqrt{(-3)^2 + (-3)^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

To find the distance between the points $B(6,7)$ and $C(9,4)$

$x_1 = 6$

$x_2 = 9$

$y_1 = 7$

$y_2 = 4$

$BC = \sqrt{(6-9)^2 + (7-4)^2}$

$= \sqrt{(-3)^2 + (3)^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

To find the distance between the points $C(9,4)$ and $(6,1)$

$x_1 = 9$

$x_2 = 6$

$y_1 = 4$

$y_2 = 1$

$CD = \sqrt{(9-6)^2 + (4-1)^2}$

$= \sqrt{(-3)^2 + (3)^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$ To find the distance between the points $A(3, 4)$ and $D(6,1)$

$x_1 = 3$

$x_2 = 6$

$y_1 = 4$

$y_2 = 1$

$AB = \sqrt{(3-6)^2 + (4-1)^2}$

$= \sqrt{(-3)^2 + (3)^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

Since all sides of the squares are equal, now find the distance between the diagonals AC and BD.

To find the distance between the points $A(3, 4)$ and $C(9, 4)$

$x_1 = 3$

$x_2 = 9$

$y_1 = 4$

$y_2 = 4$

Diagonal $AC = \sqrt{(3-9)^2 + (4-4)^2}$

$= \sqrt{(-6)^2 + (0)^2}$

$= \sqrt{36+0}$

$= 6$

Diagonal To find the distance between the points $B(6,7)$ and $D(6,1)$

$x_1 = 6$

$x_2 = 6$

$y_1 = 7$

$y_2 = 1$

Diagonal $BD = \sqrt{(6-6)^2 + (7-1)^2}$

$= \sqrt{(0)^2 + (6)^2}$

$= \sqrt{0+36}$

$= 6$

Thus, the four sides AB, BC, CD and DA are equal and its diagonals AC and BD are also equal.

$\therefore ABCD$ form a square and hence Champa was correct.

6. Name the type of quadrilateral forms, if any, by the following points, and give reasons for your answer.

(i) $(-1,-2),(1,0),(-1,2),(-3,0)$

Ans: Given that,

Let the given points denote the vertices $A(-1,-2)$, $B(1,0)$, $C(-1,\ 2)$, $D(-3,0)$ denote the vertices of the quadrilateral.

The distance between any two points is given by the Distance formula,

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

To find the distance between the points $A(-1,-2)$ and $B(1,0)$

$x_1 = -1$

$x_2 = 1$

$y_1 = -2$

$y_2 = 0$

$AB = \sqrt{(-1-1)^2 + (-2-0)^2}$

$= \sqrt{(-2)^2 + (-2)^2}$

$= \sqrt{4+4}$

$= \sqrt{8}$

$= 2\sqrt{2}$

To find the distance between the points $B(1,0)$ and $C(-1,2)$

$x_1 = 1$

$x_2 = -1$

$y_1 = 0$

$y_2 = 2$

$BC = \sqrt{(1-(-1))^2 + (0-2)^2}$

$= \sqrt{(2)^2 + (-2)^2}$

$= \sqrt{4+4}$

$= \sqrt{8}$

$= 2\sqrt{2}$

To find the distance between the points $C(-1,2)$ and $D(-3,0)$

$x_1 = -1$

$x_2 = -3$

$y_1 = 2$

$y_2 = 0$

$CD = \sqrt{(-1-(-3))^2 + (2-0)^2}$

$= \sqrt{(2)^2 + (2)^2}$

$= \sqrt{4+4}$

$= \sqrt{8}$

$= 2\sqrt{2}$

To find the distance between the points $D(-3,0)$ and $A(-1,-2)$

$x_1 = -3$

$x_2 = -1$

$y_1 = 0$

$y_2 = -2$

$AD = \sqrt{\left(-3-(-1)\right)^2 + \left(0-(-2)\right)^2}$

$= \sqrt{(-2)^2 + (2)^2}$

$= \sqrt{4+4}$

$= \sqrt{8}$

$= 2\sqrt{2}$

To find the diagonals of the given quadrilateral

To find the distance between the points $A(-1,-2)$ and $C(-1, 2)$

$x_1 = -1$

$x_2 = -1$

$y_1 = -2$

$y_2 = 2$

Diagonal $AC = \sqrt{\left(-1-(-1)\right)^2 + \left(-2-2\right)^2}$

$= \sqrt{(0)^2 + (-4)^2}$

$= \sqrt{0+16}$

$= 4$

To find the distance between the points $B(1,0)$ and $D(-3,0)$

$x_1 = 1$

$x_2 = -3$

$y_1 = 0$

$y_2 = 0$

Diagonal $BD = \sqrt{\left(1-(-3)\right)^2 + \left(0-0\right)^2}$

$= \sqrt{(4)^2 + (0)^2}$

$= \sqrt{16+0}$

$= 4$

Since all sides of the given quadrilateral are of the same measure and the diagonals are also the same length.

$\therefore$ The given points of the quadrilateral form a square.

(ii) $\mathbf{(-3,5),(3,1),(0,3),(-1,-4)}$

Ans: Given that,

Let the given points denote the vertices $A(-3,5)$, $B(3,1)$,$C(0,3)$, $D(-1,\ -4)$ denote the vertices of the quadrilateral.

The distance between any two points is given by the Distance formula,

$$d=\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}$$

To find the distance between the points $A(-3,5)$ and $B(3,1)$

$x_1=-3$

$x_2=3$

$y_1=5$

$y_2=1$

$$AB=\sqrt{\left(-3-3\right)^2+\left(5-1\right)^2}$$

$$=\sqrt{\left(-6\right)^2+\left(4\right)^2}$$

$$=\sqrt{36+16}$$

$$=\sqrt{52}$$

$$=2\sqrt{13}$$

To find the distance between the points $B(3,1)$ and $C(0,3)$

$x_1=3$

$x_2=0$

$y_1=1$

$y_2=3$

$$BC=\sqrt{\left(3-0\right)^2+\left(1-3\right)^2}$$

$= \sqrt{(3)^2 + (-2)^2}$

$= \sqrt{9+4}$

$= \sqrt{13}$

To find the distance between the points $C(0,3)$ and $D(-1,-4)$

$x_1 = 0$

$x_2 = -1$

$y_1 = 3$

$y_2 = -4$

$CD = \sqrt{(0-(-1))^2 + (3-(-4))^2}$

$= \sqrt{(1)^2 + (7)^2}$

$= \sqrt{1+49}$

$= \sqrt{50}$

$= 5\sqrt{2}$

To find the distance between the points $A(-3,5)$ and $D(-1,-4)$

$x_1 = -3$

$x_2 = -1$

$y_1 = 5$

$y_2 = -4$

$AD = \sqrt{(-3-(-1))^2 + (5-(-4))^2}$

$= \sqrt{(-2)^2 + (9)^2}$

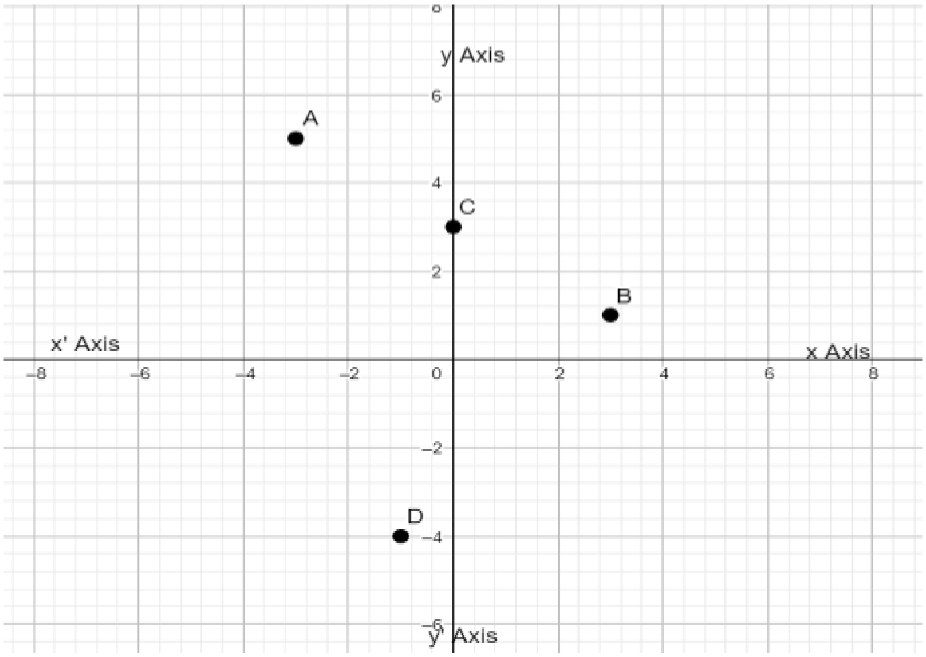
$= \sqrt{4+81}$

$= \sqrt{85}$

From the distance we found that,

$AB \neq BC \neq CD \neq AD$

By plotting the graph, we get,



From the graph above, we can note that the points ABC are collinear.

$\therefore$ The quadrilateral cannot be formed by using the above points.

(iii) $\mathbf{(4,5),(7,6),(4,3),(1,2)}$

Ans: Given that,

Let the given points denote the vertices $A(4,5), B(7,6), C(4,3), D(1,2)$ denote the vertices of the quadrilateral.

The distance between any two points is given by the Distance formula,

$$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

To find the distance between the points $A(4,5)$ and $B(7,6)$

$x_1=4$

$x_2=7$

$y_1=5$

$y_2=6$

$$AB=\sqrt{(4-7)^2+(5-6)^2}$$

$$=\sqrt{(-3)^2+(-1)^2}$$

$= \sqrt{9+1}$

$= \sqrt{10}$

To find the distance between the points $B(7,6)$ and $C(4,3)$

$x_1 = 7$

$x_2 = 4$

$y_1 = 6$

$y_2 = 3$

$BC = \sqrt{(7-4)^2 + (6-3)^2}$

$= \sqrt{(3)^2 + (3)^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

To find the distance between the points $C(4,3)$ and $D(1,2)$

$x_1 = 4$

$x_2 = 1$

$y_1 = 3$

$y_2 = 2$

$CD = \sqrt{(4-1)^2 + (3-2)^2}$

$= \sqrt{(3)^2 + (1)^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$

To find the distance between the points $A(4,5)$ and $D(1,2)$

$x_1 = 4$

$x_2 = 1$

$y_1 = 5$

$y_2 = 2$

$AD = \sqrt{(4-1)^2 + (5-2)^2}$

$= \sqrt{(3)^2 + (3)^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

To find the diagonals of the given quadrilateral

To find the distance between the points $A(4,5)$ and $C(4,3)$

$x_1 = 4$

$x_2 = 4$

$y_1 = 5$

$y_2 = 3$

Diagonal $AC = \sqrt{(4-4)^2 + (5-3)^2}$

$= \sqrt{(0)^2 + (2)^2}$

$= \sqrt{0+4}$

$= 2$

To find the distance between the points $B(7,6)$ and $D(1,2)$

$x_1 = 7$

$x_2 = 1$

$y_1 = 6$

$y_2 = 2$

Diagonal $BD = \sqrt{(7-1)^2 + (6-2)^2}$

$= \sqrt{(6)^2 + (4)^2}$

$= \sqrt{36+16}$

$= \sqrt{52}$

$= 2\sqrt{13}$

Form the above calculation, the opposite sides of the quadrilateral are of same length and the diagonals are not of same length

$\therefore$ The given points of the quadrilateral form a parallelogram.

7. Find the point on the x-axis which is equidistant from $(2,-5)$ and $(-2,9)$.

Ans: Given that,

$(2,-5)$

$(-2,9)$

To find,

The point that is equidistant from the points $(2,-5)$ and $(-2,9)$

Let us consider the points as $A(2,-5)$ and $B(-2,9)$ and to find the equidistant point P.

Since the point is on x-axis, the coordinates of the required point is of the form $P(x,0)$.

The distance between any two points is given by the Distance formula,

$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

To find the distance between the points $P(x,0)$ and $A(2,-5)$

$x_1=x$

$x_2=2$

$y_1=0$

$y_2=-5$

$PA=\sqrt{(x-2)^2+(0-(-5))^2}$

$=\sqrt{(x-2)^2+25}$

To find the distance between the points $A(x,0)$ and $B(-2,9)$

$x_1=x$

$x_2=-2$

$y_1=0$

$y_2 = 9$

$PB = \sqrt{(x-(-2))^2 + (0-9)^2}$

$= \sqrt{(x+2)^2 + (-9)^2}$

$= \sqrt{(x+2)^2 + 81}$

Since the distance are equal in measure,

$PA = PB$

$\sqrt{(x-2)^2 + 25} = \sqrt{(x+2)^2 + 81}$

Taking square on both sides,

$(x-2)^2 + 25 = (x+2)^2 + 81$

By solving, we get,

$x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81$

$8x = -56$

$x = -7$

The coordinate is $(-7, 0)$.

$\therefore$ The point that is equidistant from $(2, -5)$ and $(-2, 9)$ is $(-7, 0)$.

8. Find the values of y for which the distance between the points $P(2, -3)$ and $Q(10, y)$ is 10 units.

Ans: Given that,

$P(2, -3)$

$Q(10, y)$

The distance between these points are 10 units.

The distance between any two points is given by the Distance formula,

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

To find the distance between the points $P(2, -3)$ and $Q(10, y)$

$x_1 = 2$

$x_2 = 10$

$y_1 = -3$

$y_2 = y$

$PQ = \sqrt{(10-2)^2 + (y-(-3))^2}$

$= \sqrt{(-8)^2 + (y+3)^2}$

$= \sqrt{64 + (y+3)^2}$

Since the distance between them is 10 units,

$\sqrt{64 + (y+3)^2} = 10$

Squaring on both sides, we get,

$64 + (y+3)^2 = 100$

$(y+3)^2 = 36$

$y + 3 = \pm 6$

So, $y + 3 = 6$

$y = 3$

And, $y + 3 = -6$

$y = -9$

$\therefore$ The possible values of y are $y = 3$ or $y = -9$.

9. If $Q(0,1)$ is equidistant from $P(5,-3)$ and $R(x,6)$, find the values of x. Also find the distance of QR and PR.

Ans: Given that,

$Q(0,1)$

$P(5,-3)$

$R(x,6)$

To find,

- The values of x
- The distance of QR and PR

Q is equidistant between P and R.

SO $PQ = QR$

The distance between any two points is given by the Distance formula,

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\sqrt{(5-0)^2 + (-3-1)^2} = \sqrt{(0-x)^2 + (1-6)^2}$

$\sqrt{(5)^2 + (-4)^2} = \sqrt{(-x)^2 + (-5)^2}$

$\sqrt{25+16} = \sqrt{x^2 + 25}$

Squaring on both sides, we get,

$41 = x^2 + 25$

$x^2 = 16$

$x = \pm 4$

Thus the point R is $R(4,6)$ or $(-4,6)$.

To find the distance PR and QR

Case (1):

When the point is $R(4,6)$

Distance between the point $P(5,-3)$ and $R(4,6)$ is,

$PR = \sqrt{(5-4)^2 + (-3-6)^2}$

$= \sqrt{(1)^2 + (-9)^2}$

$= \sqrt{1+81}$

$= \sqrt{82}$

Distance between the point $Q(0,1)$ and $R(4,6)$,

$QR = \sqrt{(0-4)^2 + (1-6)^2}$

$= \sqrt{(-4)^2 + (-5)^2}$

$= \sqrt{16+25}$

$= \sqrt{41}$

Case (2):

When the point is $R(-4,6)$

Distance between the point $P(5,-3)$ and $R(4,6)$,

$$PR=\sqrt{(5-(-4))^2+(-3-6)^2}$$
$$=\sqrt{(9)^2+(-9)^2}$$
$$=\sqrt{81+81}$$
$$=\sqrt{162}$$
$$=9\sqrt{2}$$

Distance between the point $Q(0,1)$ and $R(4,6)$,

$$QR=\sqrt{(0-(-4))^2+(1-6)^2}$$
$$=\sqrt{(4)^3+(-5)^2}$$
$$=\sqrt{16+25}$$
$$=\sqrt{41}$$

10. Find a relation between x and y such that the point (x,y) is equidistant from the point $(3,6)$ and $(-3,4)$.

Ans: Given that,

(x,y) is equidistant from $(3,6)$ and $(-3,4)$

To find,

The value of (x,y)

Let $P(x,y)$ is equidistant from $A(3,6)$ and $B(-3,4)$

Since they are equidistant,

$$PA=PB$$
$$\sqrt{(x-3)^2+(y-6)^2}=\sqrt{\left(x-(-3)^2\right)+(y-4)^2}$$
$$\sqrt{(x-3)^2+(y-6)^2}=\sqrt{(x+3)^2+(y-4)^2}$$

Squaring on both sides, we get,

$$(x-3)^2+(y-6)^2=(x+3)^2+(y-4)^2$$
$$x^2+9-6x+y^2+36-12y=x^2+9+6x+y^2+16-8y$$
$$36-16=12x+4y$$
$$3x+y=5$$
$$3x+y-5=0$$
$\therefore$ The relation between x and y is given by $3x+y-5=0$

Exercise 7.2

1. Find the coordinates of the point which divides the join of $(-1,7)$ and $(4,-3)$ in the ratio $2:3$

Ans: Given that,

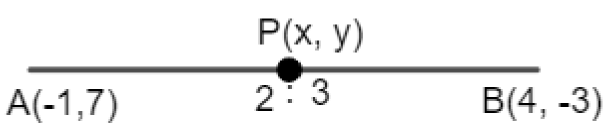
The points $A(-1,7)$ and $B(4,-3)$

Ratio $m:n=2:3$

To find,

The coordinates

Let $P(x, y)$ be the required coordinate



By section formula,

$$P(x, y)=\left[\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right]$$
$$P(x, y)=\left[\frac{2(4)+3(-1)}{2+3}, \frac{2(-3)+3(7)}{2+3}\right]$$
$$=\left[\frac{8-3}{5}, \frac{-6+21}{5}\right]$$

$= \left[\frac{5}{5}, \frac{15}{5}\right]$

$= (1,3)$

$\therefore$ The coordinates of P is $P(1,3)$.

2. Find the coordinates of the point of trisection of the line segment joining $(4,-1)$ and $(-2,-3)$.

Ans: Given that,

The line segment joining $(4,-1)$ and $(-2,-3)$

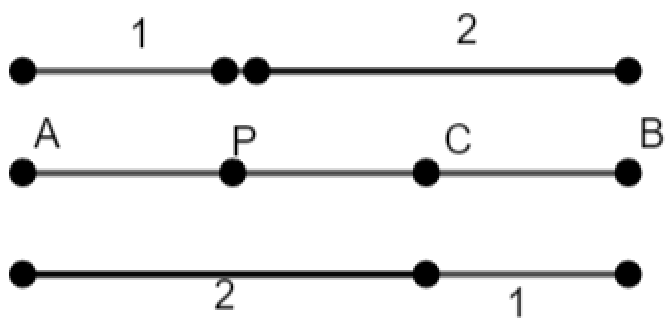
To find,

The coordinates of the point

Let the line segment joining the points be $A(4,-1)$ and $B(-2,-3)$

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ are the points of trisection of the line segment joining the points

$AP = PQ = QB$



From the diagram, the point P divides AB internally in the ratio of $1:2$

Hence $m:n = 1:2$

By section formula,

$$P(x, y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$$

$$P(x^1, y^1) = \left[\frac{1(-2)+1(4)}{1+2}, \frac{1(-3)+2(-1)}{1+2}\right]$$

$$= \left[\frac{-2+8}{3}, \frac{-3-2}{3}\right]$$

$$= \left[\frac{6}{3}, \frac{-5}{3}\right]$$

$$= \left(2, -\frac{5}{3}\right)$$

$\therefore$ The coordinates of P is $P\left(2, -\frac{5}{3}\right)$.

From the diagram, the point Q divides AB internally in the ratio of $2:1$

Hence $m:n = 2:1$

By section formula,

$$Q(x_2, y_2) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$$

$$Q(x_2, y_2) = \left[\frac{2(-2)+1(4)}{2+1}, \frac{2(-3)+3(-1)}{2+1}\right]$$

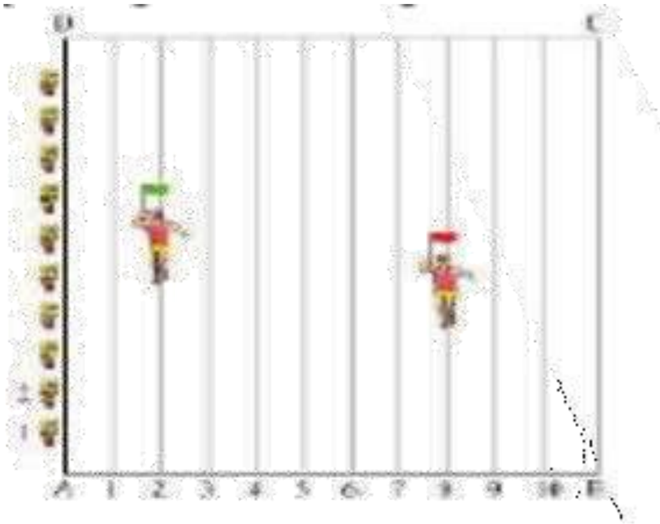
$$= \left[\frac{-4+4}{3}, \frac{-6-1}{3}\right]$$

$$= \left[0, -\frac{7}{3}\right]$$

$\therefore$ The coordinates of Q is $Q\left(0, -\frac{7}{3}\right)$.

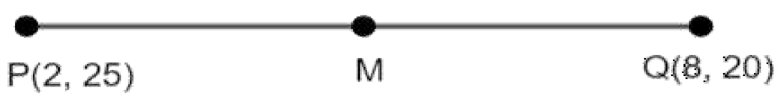
3. To conduct Sports day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in the following figure. Niharika runs $\frac{1}{4}$ th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$ ththe distance AD on the eighth line and posts a red flag. What is the distance between both the flags?If

Rashmi has to post a blue flag exactly halfway between the line segments joining the two flags, where should she post her flag?



Ans: Given that,

- Niharika posted her green flag at a distance of P which is $\frac{1}{4}\times 100 = 25\text{ m}$ from the starting point of the second line. The coordinate of P is $P(2,25)$
- Preet posted red flag at $\frac{1}{5}$ of a distance Q which is $\frac{1}{5}\times 100 = 20\text{ m}$ from the starting point of the eighth line. The coordinate of Q is $(8,20)$



The distance between any two points is given by the Distance formula,

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

The distance between the flags $P(2, 25), Q(8, 20)$ is given by,

$$PQ = \sqrt{(8-2)^2 + (25-20)^2}$$

$$= \sqrt{(6)^2 + (5)^2}$$

$$= \sqrt{36+25}$$

$= \sqrt{61}$ m

Rashmi should post her blue flag in the mid-point of the line joining points P and Q. Let this point be $M(x, y)$.

By section formula,

$$M(x, y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$$

$$M(x, y) = \left[\frac{1(2) + 1(8)}{1+1}, \frac{1(25) + 1(20)}{1+1}\right]$$

$$= \left[\frac{2+8}{2}, \frac{25+20}{2}\right]$$

$$= \left[\frac{10}{2}, \frac{45}{2}\right]$$

$$= (5, 22.5)$$

$\therefore$ Rashmi should place her blue flag at 22.5 m in the fifth line.

4. Find the ratio in which the line segment joining the points $(-3,10)$ and $(6,-8)$ is divided by $(-1,6)$

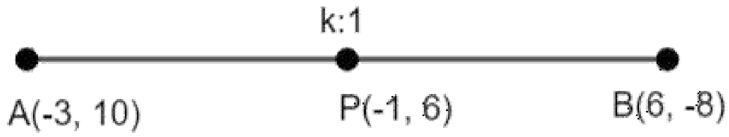
Ans: Given that,

The line segment $(-3,10)$ and $(6,-8)$

The point $(-1,6)$ divides the line segment

To find,

The ratio of dividing line segment



Let the line segment be $A(-3,10)$ and $B(6,-8)$ divided by point $P(-1,6)$ in the ratio of $k:1$

By section formula,

$$P(x, y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$$

Equating the x term,

$$-1 = \frac{6k-3}{k+1}$$

$$-k - 1 = 6k - 3$$

$$7k = 2$$

$$k = \frac{2}{7}$$

$\therefore$ The point P divides the line segment AB in the ratio of $2:7$

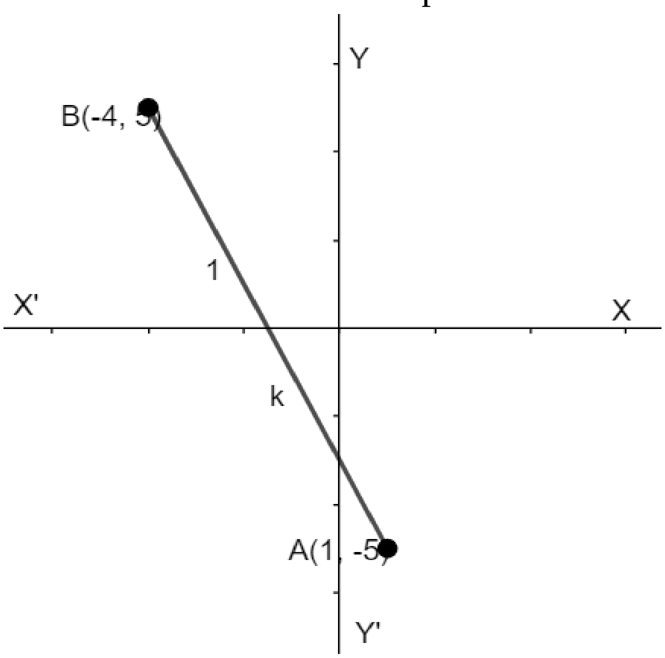
5. Find the ratio in which the line segment joining $A(1,-5)$ and $B(-4,5)$ is divided by the x axis. Also find the coordinates of the point of division.

Ans: Given that,

The line segment joining the points $A(1,-5)$ and $B(-4,5)$

To find,

- The ratio
- The coordinates of the point of division



Let the ratio be $k:1$

By section formula,

$$P(x, y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$$

$$P(x, y) = \left[\frac{k(-4) + 1(1)}{k+1}, \frac{k(5) + 1(-5)}{k+1}\right]$$

$$= \left[\frac{-4k+1}{k+1}, \frac{5k-5}{k+1}\right]$$

We know that y coordinate on x axis is zero.

$$\frac{5k-5}{k+1} = 0$$

$$5k - 5 = 0$$

$$5k = 5$$

$$k = 1$$

Therefore x axis divides it in the ratio of 1:1

Division point, $P = \left(\frac{-4(1)+1}{1+1}, \frac{5(1)-5}{1+1}\right)$

$$= \left(\frac{-4+1}{2}, \frac{5-5}{2}\right)$$

$$= \left(-\frac{3}{2}, 0\right)$$

$\therefore$ The ratio at which the line segment is divided is $1:1$ and the point of division is $\left(-\frac{3}{2}, 0\right)$.

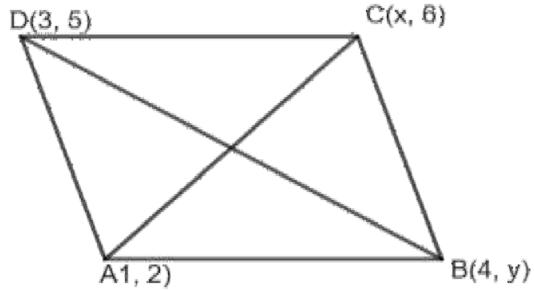
6. If $(1,2),(4,y),(x,6)$ and $(3,5)$ are the vertices of the parallelogram taken in order, find x and y.

Ans: Given that,

The vertices of the parallelogram are $A(1,2), B(4,y), C(x,6), D(3,5)$

To find,

The value of x and y



- The diagonals of the parallelogram bisect each other at O.
- Intersection point O of diagonal AC and BD divides these diagonals. So O is the midpoint of AC and BD.

If O is the midpoint of AC,

$$O = \left(\frac{1+x}{2}, \frac{2+6}{2}\right)$$

$$= \left(\frac{1+x}{2}, \frac{8}{2}\right)$$

$$= \left(\frac{1+x}{2}, 4\right)$$

If O is the midpoint of BD,

$$O = \left(\frac{4+3}{2}, \frac{5+y}{2}\right)$$

$$= \left(\frac{7}{2}, \frac{5+y}{2}\right)$$

Equating the points of O,

$$\frac{x+1}{2} = \frac{7}{2}$$ and

$$4 = \frac{5+y}{2}$$

Finding x term,

$$\frac{x+1}{2} = \frac{7}{2}$$

$x+1=7$

$x=6$

Finding y term,

$4=\frac{5+y}{2}$

$5+y=8$

$y=3$

The value of x and y are $x=6$ and $y=3$

7. Find the coordinates of point A, where AB is the diameter of circle whose center is $(2,-3)$ and B is $(1,4)$

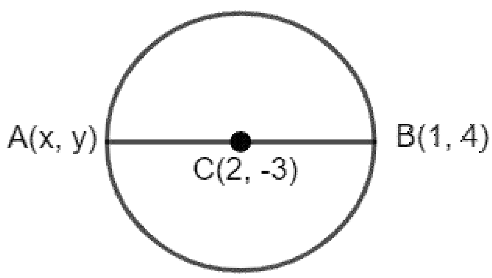
Ans: Given that,

- Center is $C(2,-3)$
- The coordinate of B is $B(1,4)$

To find,

The coordinate of A

Let the coordinate of A be $A(x,y)$



Midpoint of AB is $C(2,-3)$ and so,

$(2,-3)=\left(\frac{x+1}{2},\frac{y+4}{2}\right)$

Equating x term,

$\frac{x+1}{2}=2$

$x+1=4$
$x=3$
Equating y term,

$\frac{y+4}{2}=-3$

$y+4=-6$

$y=-10$

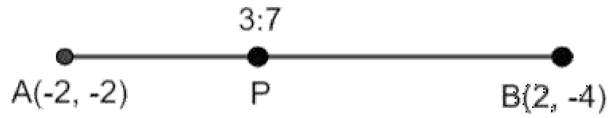
$\therefore$ The coordinate of A is $A(3,-10)$

8. If A and B are $(-2,-2)$ and $(2,-4)$ respectively, find the coordinates of P such that $AP=\frac{3}{7}AB$ and P lies on the line segment AB.

Ans: Given that,

- The coordinates are $A(-2,-2)$ and $B(2,-4)$
- $AP=\frac{3}{7}AB$

To find,
The coordinate of P



$AP=\frac{3}{7}AB$

$\frac{AB}{AP}=\frac{7}{3}$

From the figure, $AB=AP+PB$

$\frac{AP+PB}{AP}=\frac{3+4}{3}$

$1+\frac{PB}{AP}=1+\frac{4}{3}$

$$\frac{PB}{AP}=\frac{4}{3}$$

$\therefore AP:PB=3:4$

Thus, the point $P(x,y)$ divides the line segment AB in the ratio of $3:4$

By section formula,

$$P(x,y)=\left[\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right]$$

$$P(x,y)=\left[\frac{3(2)+4(-2)}{3+4},\frac{3(-4)+4(-2)}{3+4}\right]$$

$$=\left[\frac{6-8}{7},\frac{-12-8}{7}\right]$$

$$=\left[-\frac{2}{7},-\frac{20}{7}\right]$$

$\therefore$ The coordinates of P is $P\left(-\frac{2}{7},-\frac{20}{7}\right)$.

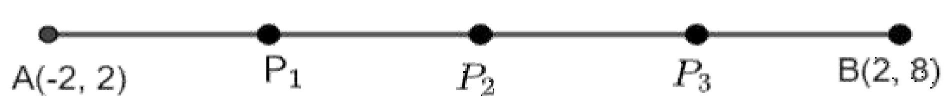
7. Find the coordinates of the points which divide the line segment joining $A(-2,2)$ and $B(2,8)$ into four equal parts.

Ans: Given that,

The line segment $A(-2,2)$ and $B(2,8)$

To find,

The coordinate that divides the line segment into four equal parts



Form the figure, P_1,P_2,P_3 be the points that divide the line segment AB into four equal parts.

Point P_1 divides the line segment AB in the ratio of $1:3$, so,

By section formula,

$$P(x, y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$$

$$P_1(x, y) = \left[\frac{1(2) + 3(-2)}{1+3}, \frac{1(8) + 3(2)}{1+3}\right]$$

$$= \left[\frac{-4}{4}, \frac{14}{4}\right]$$

$$= \left(-1, \frac{7}{2}\right)$$

Point P_2 divides the line segment AB in the ratio of 1:1, so,

By section formula,

$$P(x, y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$$

$$P_2(x, y) = \left[\frac{1(2) + 1(-2)}{1+1}, \frac{1(8) + 1(2)}{1+1}\right]$$

$$= \left[\frac{2 + (-2)}{2}, \frac{2+8}{2}\right]$$

$$= (0,5)$$

Point P_3 divides the line segment AB in the ratio of $3:1$, so,

By section formula,

$$P(x, y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$$

$$P_3(x, y) = \left[\frac{3(2) + 1(-2)}{3+1}, \frac{3(8) + 1(2)}{3+1}\right]$$

$$= \left[\frac{6-2}{4}, \frac{24+2}{4}\right]$$

$$= \left(1, \frac{13}{2}\right)$$

The coordinates that divide the line segment into four equal parts are $P_1\left(-1,\frac{7}{2}\right), P_2(0,5)$ and $P_3\left(1,\frac{13}{2}\right)$

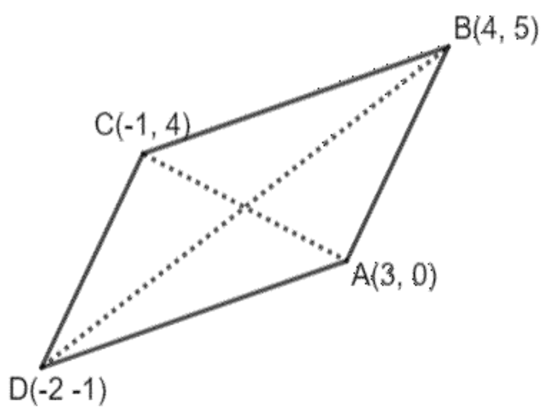
10. Find the area of the rhombus if its vertices are $(3,0),(4,5),(-1,4)$ and $(-2,-1)$ taken in order. [Hint: Area of a rhombus = $\frac{1}{2}$(Product of its diagonals)]

Ans: Given that,

The vertices of the rhombus are $A(3,0), B(4,5), C(-1,4)$ and $D(-2,-1)$

To find,

The area of the rhombus



The distance between any two points is given by the Distance formula,

$$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

Distance between the diagonal AC is given by,

$$AC=\sqrt{(3-(-1))^2+(0-4)^2}$$

$$=\sqrt{(4)^2+(-4)^2}$$

$$=\sqrt{16+16}$$

$$=\sqrt{32}$$

$= 4\sqrt{2}$

Distance between the diagonal BD si given by,

$$BD = \sqrt{(4-(-2))^2 + (5-(-1))^2}$$

$$= \sqrt{(6)^2 + (6)^2}$$

$$= \sqrt{36+36}$$

$$= \sqrt{72}$$

$$= 6\sqrt{2}$$

Area of the rhombus = $\frac{1}{2}\times$(Products of lengths of diagonals

$$= \frac{1}{2} \times AC \times BD$$

$$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$$

$= 24$square units

Exercise 7.3

1. Find the area of the triangle whose vertices are

(i) $(2,3),(-1,0),(2,-4)$

Ans: Given that,

The vertices of the triangle whose points are $A(2,3), B(-1,0), C(2, \ -4)$

To find,

The area of the triangle

Area of the triangle $= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

Substitute the value of x_1, x_2, x_3 in the above formula,

Area of given triangle $= \frac{1}{2}\left[2(0-(-4)) + -1(-4-3) + 2(3-0)\right]$

$= \frac{1}{2}(8 + 7 + 6)$

$= \frac{21}{2}$ square units

Area of the triangle with vertices $A(2,3), B(-1,0), C(2, \ -4)$ is $\frac{21}{2}$ square units

(ii) $\mathbf{(-5,-1),(3,-5),(5,2)}$

Ans: Given that,

The vertices of the triangle whose points are $A(-5, -1), B(3, -5), C(5, 2)$

To find,

The area of the triangle

Area of the triangle $= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

Substitute the value of x_1, x_2, x_3 in the above formula,

Area of given triangle $= \frac{1}{2}\left[-5(-5-(-2)) + 3(2-(-1)) + 5(-1-(-5))\right]$

$= \frac{1}{2}(35 + 9 + 20)$

$= 32$ square units

Area of the triangle with vertices $A(-5, -1), B(3, -5), C(5, 2)$ is 32 square units.

2. In each of the following find the value of 'k', for which the points are collinear.

(i) $\mathbf{(7,-2),(5,1),(3,-k)}$

Ans: Given that,

The vertices of the triangle are $A(7, \ -2), B(5,1), C(3, \ -k)$

The points are collinear

To find,

The value of k

If the points are collinear, then,

Area of the triangle $= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right] = 0$

Substitute the value of x_1, x_2, x_3 in the above equation,

$$\frac{1}{2}\left[7(1-k)+5(k-(-2))+3(-2-1)\right]=0$$
$$7-7k+5k+10-9=0$$
$$-2k+8=0$$
$$2k=8$$
$$k=4$$

$\therefore$ The value of k is 4

(ii) $\mathbf{(8,1),(k,-4),(2,-5)}$

Ans: Given that,

The vertices of the triangle are $A(8,1), B(k, -4), C(2, -5)$

The points are collinear

To find,

The value of k

If the points are collinear, then,

Area of the triangle $= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right] = 0$

Substitute the value of x_1, x_2, x_3 in the above equation,

$$\frac{1}{2}\left[8(-4-(-5))+k(-5-1)+2(1-(-4))\right]=0$$
$$8-6k+10=0$$
$$6k=18$$
$$k=3$$

$\therefore$ The value of k is 3

3. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are $(0,-1),(2,1)$ and $(0,3)$. Find the ratio of this area to the area of the given triangle.

Ans: Given that,

The vertices of the triangle are $A(0,-1)$, $B(2,1)$, $C(0,3)$

To find,

The ratio of areas of two triangles

Area of the triangle $= \frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$

Area of the triangle $ABC = \frac{1}{2}\left[0(1-3)+2(3-(-1))+0(-1-1)\right]$

$= \frac{1}{2}(8)$

$= 4$square units

A triangle is formed by joining the midpoints of the triangle ABC.

Let D,E,F be the midpoints of the sides AB,BC,CA of the triangle respectively.

$D = \left(\frac{0+2}{2}, \frac{-1+!}{2}\right)$

$= (1,0)$

$E = \left(\frac{0+0}{2}, \frac{3-1}{2}\right)$

$= (0,1)$

$F = \left(\frac{2+0}{2}, \frac{1+3}{2}\right)$

$= (1,2)$

Area of triangle $DEF = \frac{1}{2}\left[1(2-1)+1(1-0)+0(0-2)\right]$

$= \frac{1}{2}(2)$

$=1$square unit

$\therefore$ Ratio of the triangle DEF to the ratio of the triangle ABC is $1:4$

4. Find the area of the quadrilateral whose vertices, taken in order, are $(-4,-2),(-3,-5),(3,-2)$ and $(2,3)$

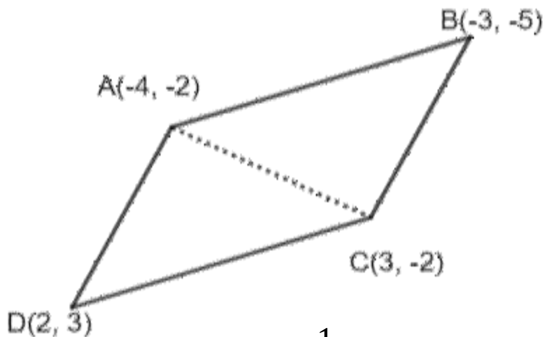
Ans: Given that,

The vertices of the triangle are $A(-4,-2), B(-3,-5), C(3,-2), D(2,3)$

To find,

The area of the quadrilateral $ABCD$

Join AC so that it forms two triangles $\triangle ABC$ and $\triangle ACD$



Area of the triangle $= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

Substituting the value of $A(-4,-2), B(-3,-5), C(3,-2)$ in the above formula,

Area of $\triangle ABC = \frac{1}{2}\left[-4(-5-(-2)) + -3(-2-(-2)) + 3(-2-(-2))\right]$

$= \frac{1}{2}[12+0+9]$

$= \frac{21}{2}$ square unit

Substituting the value of $A(-4,-2), C(3,-2), D(2,3)$ in the above formula,

Area of $\triangle ACD = \frac{1}{2}\left[-4(-2-(-3)) + 3(3-(-2)) + 2(-2-(-2))\right]$

$= \frac{1}{2}[20+15+0]$

$= \frac{35}{2}$ square unit

Area of quadrilateral $ABCD =$ Area of $\triangle ABC +$ Area of $\triangle ACD$

$= \frac{21}{2} + \frac{35}{2}$

$= \frac{56}{2}$

$= 28$ square unit

The area of the quadrilateral ABCD is 28 square units.

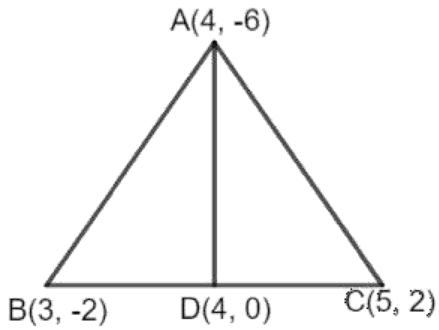
5. You have studied in Class IX that a median of a triangle divides it into two triangles of equal areas. Verify this result for $\triangle ABC$ whose vertices are $A(4,-6), B(3,-2)$ and $C(5,2)$

Ans: Given that,

The vertices of the triangle are $A(4,6)$, $B(3,-2)$, $C(5,2)$

To verify,

The median of the triangle divides the triangle into two triangles of equal areas.



Let D be the midpoint of BC.

$D = \left(\frac{3+5}{2}, \frac{-2+2}{2}\right)$

$= \left(\frac{8}{2}, 0\right)$

$= (4,0)$

AD is the median of the $\triangle ABC$ and it divides the triangle into two triangles.

Area of the triangle $= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

Substituting the values of $A(4,6), B(3,-2)$ and $D(4,0)$ in the above formula, Area of $\triangle ABD = \frac{1}{2}\left[4(-2-0)+3(0-(-6))+(-6-(-2))\right]$

$= \frac{1}{2}[-8+18-16]$

$= \frac{1}{2}(6)$

$= 3$square unit

Substituting the values of $A(4,6), D(4,0), C(5,2)$ in the above formula,

Area of $\triangle ADC = \frac{1}{2}\left[4(0-(-2))+4(2-(-6))+5(-6-0)\right]$

$= \frac{1}{2}(-8+32-30)$

$= -3$ square unit

Area cannot be in negative value. So ignore the negative value.

Both the areas are equal.

$\therefore$ Median AD of the triangle divides the triangle into two triangles of equal areas

Exercise 7.4

1. Determine the ratio in which the line $2x+y-4=0$ divides the line segment joining the points $A(2,-2)$ and $B(3,7)$.

Ans: Given that,

- A line $2x+y-4=0$
- A line segment joining the point $A(2,-2)$ and $B(3,7)$

To determine,

The ratio

The given line $2x+y-4=0$ divides the line segment joining the points $A(2,-2)$ and $B(3,7)$ in a ratio of $k:1$ at the point C

By section formula,

$$P(x, y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$$

$$C(x, y) = \left[\frac{k(3) + 1(2)}{k+1}, \frac{k(7) + 1(-2)}{k+1}\right]$$

$$= \left[\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right]$$

The point C also lies on the line $2x + y - 4 = 0$

Substituting the value of $C(x, y)$ in the given line, we get,

$$2\left(\frac{3k+2}{k+1}\right) + \left(\frac{7k-2}{k+1}\right) - 4 = 0$$

By solving this, we get,

$$\frac{6k + 4 + 7k - 2 - 4k - 4}{k+1} = 0$$

$$9k - 2 = 0$$

$$k = \frac{2}{9}$$

$\therefore$ The ratio in which the line $2x + y - 4 = 0$ divides the line joining two points $A(2, -2)$ and $B(3,7)$ is $2:9$ internally.

2. Find a relation between x and y if the points $(x, y), (1, 2)$ and $(7, 0)$ are collinear.

Ans: Given that,

The points $A(x, y), B(1, 2), C(7, 0)$

The points are collinear

To find,

The relation between x and y

Since the given points are collinear, then the area of the triangle formed by these points is equal to zero.

Area of the triangle $= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

Substituting the above points in the above formula, w get,

Area $=\frac{1}{2}\left[x(2-0)+1(0-y)+7(y-2)\right]$

$\frac{1}{2}[2x-y+7y-14]=0$

$2x+6y-14=0$

$x+3y-7=0$

$\therefore$ The relation between x and y is $x+3y-7=0$

3. Find the center of the circle passing through the points $(6,-6),(3,-7)$ and $(3,3)$.

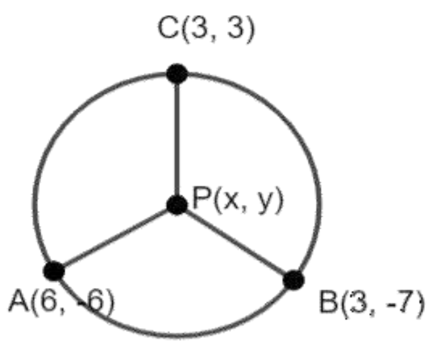
Ans: Given that,

Center passing through the points $A(6,\ -6),B(3,\ -7),C(3,3)$

To find,

The center of the circle

Let $O(x,y)$ be the center of the circle



Distance from center O to the points A,B,C is the same since they form the radius of the circle.

The distance between any two points is given by the Distance formula,

$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$OA=\sqrt{(x-6)^2+(y+6)^2}$

$\text{OB} = \sqrt{(x-3)^2 + (y+7)^2}$

$\text{OC} = \sqrt{(x-3)^2 + (y-3)^2}$

Equate $\text{OA} = \text{OB}$ since it represents radius

$\sqrt{(x-6)^2 + (y+6)^2} = \sqrt{(x-3)^2 + (y+7)^2}$

Squaring on both sides and solving them, we get,

$x^2 + 36 - 12x + y^2 + 36 + 12y = x^2 + 9 - 6x + y^2 + 49 + 14y$

$-6x - 2y + 14 = 0$

$3x + y = 7 \qquad \ldots\ldots(1)$

Similarly, $\text{OA} = \text{OC}$ since it represents radius

$\sqrt{(x-6)^2 + (y+6)^2} = \sqrt{(x-3)^2 + (y-3)^2}$

Squaring on both sides and solving them, we get,

$x^2 + 36 - 12x + y^2 + 36 + 12y = x^2 + 9 - 6x + y^2 + 9 - 6y$

$-6x + 18y + 54 = 0$

$-3x + 9y = -27 \qquad \ldots\ldots(2)$

By adding (1) and (2), w get,

$10y = -20$

$y = -2$

Substituting $y = -2$ in the equation (1), we get,

$3x - 2 = 7$

$3x = 9$

$x = 3$

$\therefore$ The center of the circle is $C(3, -2)$

4. The two opposite vertices of a square are $(-1, 2)$ and $(3, 2)$. Find the coordinates of the two other two vertices

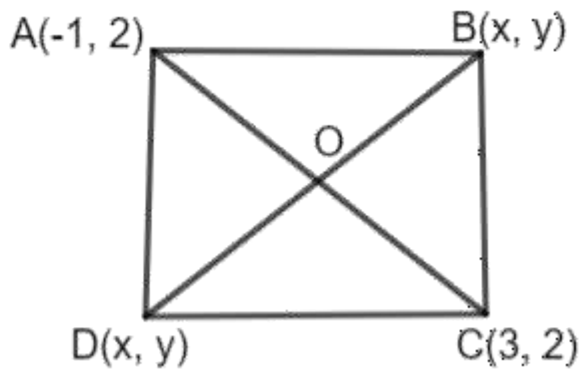
Ans: Given that,

The two opposite vertices of the square are $A(1, -2)$ and $C(3, 2)$

To find,

The other two coordinates

Let $ABCD$ be a square having the vertices $A(-1,2), B(x_1, y_1), C(3,2), D(x_2, y_2)$



We know that,

Sides of a square are equal

$AB = BC$

The distance between any two points is given by the Distance formula,

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

The distance between AB and BC is given by,

$\sqrt{(x-1)^2 + (y-2)^2} = \sqrt{(x-3)^2 + (y-2)^2}$

Squaring on both sides and solving them, we get,

$x^2 + 2x + 1 + y^2 - 4y + 4 = x^2 + 9 - 6x + y^2 + 4 - 4y$

$8x = 8$

$x = 1$

We know that,

All interior angles in a square are of angle $90°$

Consider $\triangle ABC$,

By Pythagoras theorem,

$(AB)^2 + (BC)^2 = (AC)^2$

Distance formula is used to find the distance of AB, BC, AC and substituting them in the Pythagoras theorem,

$\left(\sqrt{(1+1)^2 + (y-2)^2}\right)^2 + \left(\sqrt{(1-3)^2 + (y-2)^2}\right)^2 = \left(\sqrt{(3+1)^2 + (2-2)^2}\right)^2$

$4+y^2+4-4y+4+y^2-4y+4=16$

$2y^2+16-8y=16$

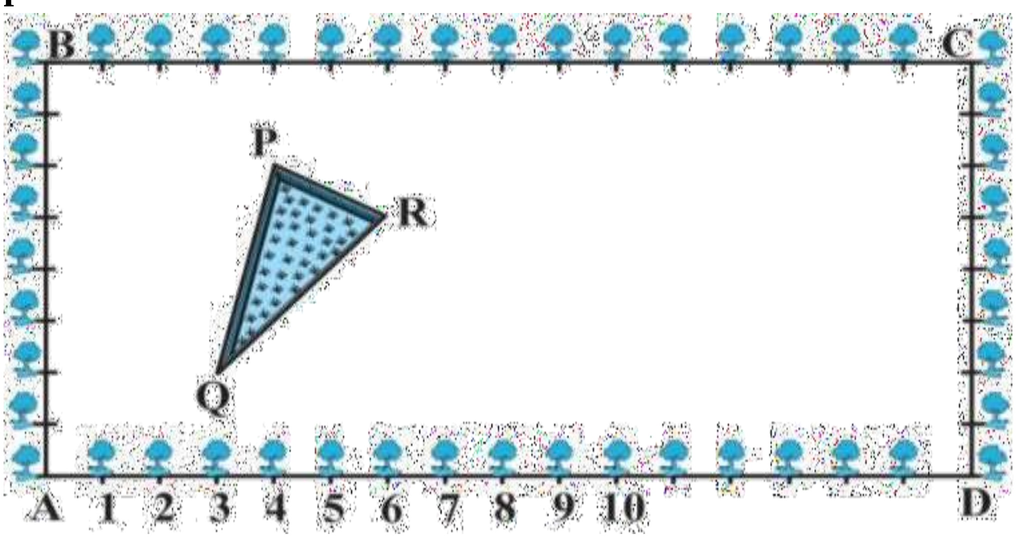
$2y^2-8y=0$

$2y(y-4)=0$

$y=0$ or 4

$\therefore$ The other two vertices are $B(1,0)$ and $D(1,4)$

5. The class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are plotted on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in the following figure. The students are to sow seeds of flowering plants on the remaining area of the plot.



(i) Taking A as origin, find the coordinates of the vertices of the triangle

Ans: Given that,

- Saplings are plotted 1 m from each other
- By taking A as origin, To find the coordinates of the vertices of the triangle
- From the diagram, it can be observed that the coordinates are $P(4,6), Q(3,2), R(6,5)$

Area of the triangle $= \frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$

Substitute the values of P,Q,R in the above formula, we get,

Area of triangle $PQR = \frac{1}{2}\left[4(2-5)+3(5-6)+6(6-2)\right]$

$= \frac{1}{2}[-12-3+24]$

$= \frac{9}{2}$ square unit

$\therefore$ The coordinates are $P(4,6), Q(3,2), R(6,5)$

(ii) What will be the coordinates of the vertices of $\triangle PQR$ if C is the origin? Also calculate the areas of the triangles in these cases. What do you observe?

Ans: Given that,

- Saplings are plotted 1 m from each other
- Taking C as origin, CB as x axis, CD as y axis
- From the diagram, it can be observed that the coordinates are $P(4,6), Q(3,2), R(6,5)$

Area of the triangle $= \frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$

Substitute the value of P,Q,R in the above formula, we get,

Area of the triangle $PQR = \frac{1}{2}\left[12(6-3)+13(3-2)+10(2-6)\right]$

$= \frac{1}{2}[36-13+40]$

$= \frac{9}{2}$ square unit

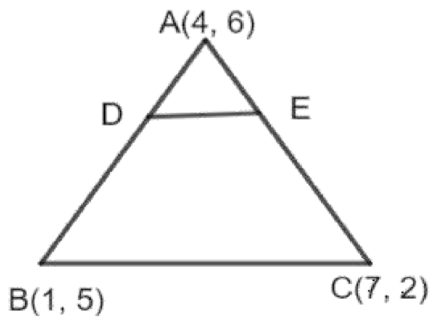
$\therefore$ The coordinates are $P(4,6), Q(3,2), R(6,5)$

The area of triangles are the same in both cases.

6. The vertices of a $\triangle ABC$ are $A(4,6), B(1,5)$ and $C(7,2)$. A line is drawn to intersect sides AB and AC at D and E respectively, such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. Calculate the area of the $\triangle ADE$ and compare it with the area of $\triangle ABC$. (Recall the converse of basic proportionality theorem and Theorem 6.6 related to Ratio of areas of two similar triangles.)

Ans: Given that,

- The vertices of $\triangle ABC$ are $A(4,6)$, $B(1,5)$, $C(7, 2)$
- $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$
- To calculate the area of $\triangle ADE$ and compare it with $\triangle ABC$



From the diagram, we observe that D and E are the mid-points of AB and AC respectively so that they divide the line segment AB and AC in the ratio of $1:3$

By section formula,

$$P(x, y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$$

$$D(x, y) = \left[\frac{1(1) + 3(4)}{1+3}, \frac{1(5) + 3(6)}{1+3}\right]$$

$$= \left[\frac{1+12}{4}, \frac{5+18}{4}\right]$$

$$= \left[\frac{13}{4}, \frac{23}{4}\right]$$

$$E(x, y) = \left[\frac{1(7)+3(4)}{1+3}, \frac{1(2)+3(6)}{1+3}\right]$$

$$= \left[\frac{7+12}{4}, \frac{2+18}{4}\right]$$

$$= \left[\frac{19}{4}, \frac{20}{4}\right]$$

Area of the triangle $= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

Area of $\triangle ADE = \frac{1}{2}\left[4\left(\frac{23}{4} - \frac{20}{4}\right) + \frac{13}{4}\left(\frac{20}{4} - 6\right) + \frac{19}{4}\left(6 - \frac{23}{4}\right)\right]$

$$= \frac{1}{2}\left[2 - \frac{13}{4} + \frac{19}{16}\right]$$

$$= \frac{1}{2}\left[\frac{67-52}{16}\right]$$

$= \frac{15}{32}$ square unit.

Area of $\triangle ABC = \frac{1}{2}\left[4(5-2) + 1(2-6) + 7(6-5)\right]$

$$= \frac{1}{2}(12 - 4 + 7)$$

$= \frac{15}{2}$ square unit

Thus the ratio of areas of $\triangle ADE$ to $\triangle ABC$ is 1:16

If a line segment in a triangle divides its two sides in the same ratio, then the line segment is parallel to the third side of the triangle. Thus these triangles are similar to each other.

And the ratio between the areas of two triangles will be equal to the square of the ratio between the sides of the triangle.

$\therefore$ The ratio between the areas of the $\triangle ADE$ and $\triangle ABC$ is 1:16

7. Let $A(4,2)$, $B(6,5)$ and $C(1,4)$ be the vertices of the $\triangle ABC$

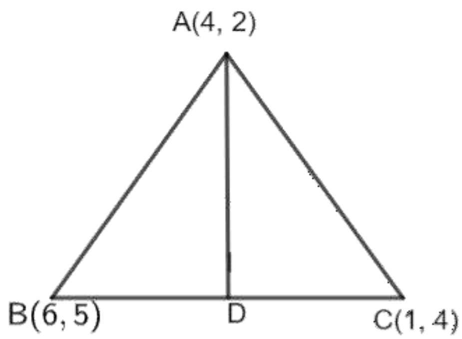
(i) The median from A meets BC at D. Find the coordinates of point D

Ans: Given that,

$A(4, 2), B(6,5), C(1, 4)$ are the vertices of the given triangle.

To find,

The coordinates of point D



Let AD be the median of the given triangle.

So median D is the midpoint of BC

So,

$$D(x, y) = \left(\frac{6+1}{2}, \frac{5+4}{2}\right)$$

$$= \left(\frac{7}{2}, \frac{9}{2}\right)$$

$\therefore$ The coordinate of D is $\left(\frac{7}{2}, \frac{9}{2}\right)$

(ii) Find the coordinates of point P on AD such that $AP : PD = 2:1$

Ans: Given that,

$A(4, 2), B(6,5), C(1, 4)$ are the vertices of the given triangle.

$AP : PD = 2:1$

To find,

The coordinate of P

Point P divides AB in the ratio of $m:n = 2:1$

By section formula,

$$P(x, y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$$

$$P(x, y) = \left[\frac{2\left(\frac{7}{2}\right) + 1(4)}{2+1}, \frac{2\left(\frac{9}{2}\right) + 1(2)}{2+1}\right]$$

$$= \left[\frac{7+4}{3}, \frac{9+2}{3}\right]$$

$$= \left(\frac{11}{3}, \frac{11}{3}\right)$$

$\therefore$ The coordinates of P is $P\left(\frac{11}{3}, \frac{11}{3}\right)$.

(iii) Find the coordinate of point Q and R on medians BE and CF respectively such that BQ: QE = 2 :1 and CR : RF = 2 :1. What do you observe?

Ans: Given that,

$A(4, 2), B(6,5), C(1, 4)$ are the vertices of the given triangle.

$BQ:QE = 2:1$

$CR:RF = 2:1$

To find,

Coordinate of Q and R

4to find the coordinate of E

Median BE of the triangle will divide the side AC in two equal parts.

So E is the midpoint of AC

$$E = \left(\frac{4+1}{2}, \frac{2+4}{2}\right)$$

$$= \left(\frac{5}{2}, \frac{6}{2}\right)$$

$$= \left(\frac{5}{2}, 3\right)$$

Q divides BE in the ratio of $2:1$

By section formula,

$$P(x, y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$$

$$Q(x, y) = \left[\frac{2\left(\frac{5}{2}\right) + 1(6)}{2+1}, \frac{2(3) + 1(5)}{2+1}\right]$$

$$= \left[\frac{5+6}{3}, \frac{6+5}{3}\right]$$

$$= \left(\frac{11}{3}, \frac{11}{3}\right)$$

Median CF divide AB in the ratio of $2:1$.

So F is the midpoint of AB

$$F = \left(\frac{4+6}{2}, \frac{2+5}{2}\right)$$

$$= \left(\frac{10}{2}, \frac{7}{2}\right)$$

$$= \left(5, \frac{7}{2}\right)$$

By section formula,

$$P(x, y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$$

$$R(x, y) = \left[\frac{2(5)+1(1)}{2+1}, \frac{2\left(\frac{7}{2}\right)+1(4)}{2+1}\right]$$

$$= \left[\frac{10+1}{3}, \frac{7+4}{3}\right]$$

$$= \left(\frac{11}{3}, \frac{11}{3}\right)$$

The coordinates of Q and that of R is $\left(\frac{11}{3}, \frac{11}{3}\right)$

(iv) What do you observe?

Ans: Given that,

$A(4, 2), B(6,5), C(1, 4)$ are the vertices of the given triangle.

Form the above observation, we found that the coordinates of P,Q,R are all same. So, all these points are representing the same point on the same plane which is the centroid of the triangle.

(v) If $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of $\triangle ABC$, find the coordinates of the centroid of the triangle.

Ans: Given that,

$A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of the $\triangle ABC$

To find,

The coordinate of the centroid of the triangle

Median AD divides the side BC into two equal parts. So D is the midpoint of BC

$$D = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$$

Let the centroid of this triangle be O and O divides the side AD in the ratio of $2:1$

By section formula,

$$P(x, y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$$

$$O(x, y) = \left[\frac{2\left(\frac{x_2 + x_3}{2}\right) + 1(x_1)}{2+1}, \frac{2\left(\frac{y_2 + y_3}{2}\right) + 1(x_1)}{2+1}\right]$$

$$= \left[\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right]$$

The centroid of the triangle ABC with vertices $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ is

$$\left[\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right]$$

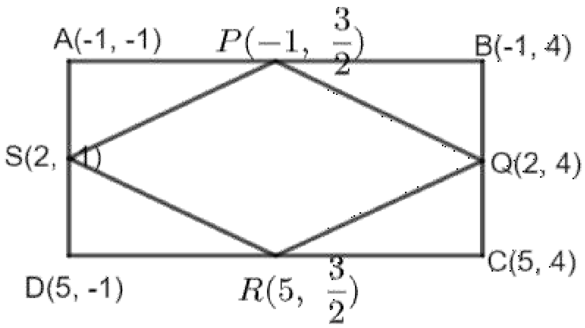
8. ABCD is a rectangle formed by the points $A(-1, -1), B(-1, 4), C(5, 4)$ and $D(5, -1)$. P,Q,R and S are the midpoints of AB,BC,CD and DA respectively. Is the quadrilateral PQRSa square? A rectangle? Or a rhombus? Justify your answer.

Ans: Given that,

$A(-1,-1), B(-1, 4), C(5, 4), D(5, -1)$ are the vertices of the rectangle.

To find,

If $PQRS$ is a square or rectangle or rhombus



Form the figure,

P is the midpoint of AB

$$P = \left(\frac{-1-1}{2}, \frac{-1+4}{2}\right)$$

$$= \left(-\frac{2}{2}, \frac{3}{2}\right)$$

$$= \left(-1, \frac{3}{2}\right)$$

$$Q = \left(\frac{-1+5}{2}, \frac{4+4}{2}\right)$$

$$= (2, 4)$$

$$R = \left(\frac{5+5}{2}, \frac{4+(-1)}{2}\right)$$

$$= \left(5, \frac{3}{2}\right)$$

$$S = \left(\frac{-1+5}{2}, \frac{-1-1}{2}\right)$$

$$= (2, -1)$$

The distance between any two points is given by the Distance formula,

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Distance between the points P and Q is,

$$PQ=\sqrt{(-1-2)^2+\left(\frac{3}{2}-4\right)^2}$$

$$=\sqrt{9+\frac{25}{4}}$$

$$=\sqrt{\frac{61}{4}}$$

Distance between the points Q and R is,

$$QR=\sqrt{(2-5)^2+\left(4-\frac{3}{2}\right)^2}$$

$$=\sqrt{9+\frac{25}{4}}$$

$$=\sqrt{\frac{61}{4}}$$

Distance between the points R and S is,

$$RS=\sqrt{(5-2)^2+\left(\frac{3}{2}+1\right)^2}$$

$$=\sqrt{9+\frac{25}{4}}$$

$$=\sqrt{\frac{61}{4}}$$

Distance between the points S and P is,

$$SP=\sqrt{(2+1)^2+\left(-1-\frac{3}{2}\right)^2}$$

$$=\sqrt{9+\frac{25}{4}}$$

$$=\sqrt{\frac{61}{4}}$$

Distance between the diagonals P and R is,

$$PR = \sqrt{(-1-5)^2 + \left(\frac{3}{2} - \frac{3}{2}\right)^2}$$

$= \sqrt{6^2}$

$= 6$

Distance between the diagonals Q and S is,

$$QS = \sqrt{(2-2)^2 + (4+1)^2}$$

$= \sqrt{5^2}$

$= 5$

From the above calculation, we observe that all sides of the quadrilateral are of the same length but the diagonals are different lengths.

$\therefore PQRS$ is a rhombus.