Maths Class 10 NCERT Solutions

Chapter 5 – Arithmetic Progression

Exercise 5.1

1. In which of the following situations, does the list of numbers involved make as arithmetic progression and why?

i) The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km.

Ans: Given the fare of first km is Rs.15 and the fare for each additional km is Rs. 8. Hence,

Taxi fare for 1^{st} km is Rs. 15.

Taxi fare for 2^{nd} km is Rs. $15+8=23$.

Taxi fare for 3^{rd} km is Rs. $23+8=31$.

Similarly, Taxi fare for n^{th} km is Rs. $15+(n-1)8$.

Therefore, we can conclude that the above list forms an A.P with common difference of 8.

ii) The amount of air present in a cylinder when a vacuum pump removes a quarter of the air remaining in the cylinder at a time.

Ans: Let the initial volume of air in a cylinder be V liter. In each stroke, the vacuum pump removes $\frac{1}{4}$ of air remaining in the cylinder at a time. Hence,

Volume after 1^{st} stroke is $\frac{3V}{4}$.

Volume after 2^{nd} stroke is $\frac{3}{4}\left(\frac{3V}{4}\right)$.

Volume after 3^{rd} stroke is $\left(\frac{3}{4}\right)^2\left(\frac{3V}{4}\right)$.

Similarly, Volume after n^{th} stroke is $\left(\frac{3}{4}\right)^n$ V.

We can observe that the subsequent terms are not added with a constant digit but are being multiplied by $\frac{3}{4}$. Therefore, we can conclude that the above list does not forms an A.P.

iii) The cost of digging a well after every meter of digging, when it costs Rs 150 for the first meter and rises by Rs 50 for each subsequent meter.

Ans: Given the cost of digging for the first meter is Rs.150 and the cost for each additional meter is Rs. 50. Hence,

Cost of digging for 1^{st} meter is Rs. 150.

Cost of digging for 2^{nd} meter is Rs. $150+50=200$.

Cost of digging for 3^{rd} meter is Rs. $200+50=250$.

Similarly, Cost of digging for n^{th} meter is Rs. $150+(n-1)50$.

Therefore, we can conclude that the above list forms an A.P with common difference of 50.

iv) The amount of money in the account every year, when Rs 10000 is deposited at compound interest at 8% per annum.

Ans: Given the principal amount is Rs.10000 and the compound interest is 8% per annum. Hence,

Amount after 1^{st} year is Rs. $10000\left(1+\frac{8}{100}\right)$.

Amount after 2^{nd} year is Rs. $10000\left(1+\frac{8}{100}\right)^2$.

Amount after 3^{rd} year is Rs. $10000\left(1+\frac{8}{100}\right)^3$.

Similarly, Amount after n^{th} year is Rs. $10000\left(1+\frac{8}{100}\right)^n$.

We can observe that the subsequent terms are not added with a constant digit but are being multiplied by $\left(1+\frac{8}{100}\right)$. Therefore, we can conclude that the above list does not forms an A.P.

2. Write first four terms of the A.P. when the first term a and the common difference d are given as follows:

(a) $a=10, d=10$

Ans: We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n - 1)d$ .....(1)

Substituting $a = 10, d = 10$ in (1) we get, $a_n = 10 + 10(n - 1) = 10n$ .....(2)

Therefore, from (2)

$a_1 = 10$, $a_2 = 20$, $a_3 = 30$ and $a_4 = 40$.

i) $\mathbf{a = -2, d = 0}$

Ans: We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n - 1)d$ .....(1)

Substituting $a = -2, d = 0$ in (1) we get, $a_n = -2 + 0(n - 1) = -2$ .....(2)

Therefore, from (2)

$a_1 = -2$, $a_2 = -2$, $a_3 = -2$ and $a_4 = -2$.

ii) $\mathbf{a = 4, d = -3}$

Ans: We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n - 1)d$ .....(1)

Substituting $a = 4, d = -3$ in (1) we get, $a_n = 4 - 3(n - 1) = 7 - 3n$ .....(2)

Therefore, from (2)

$a_1 = 4$, $a_2 = 1$, $a_3 = -2$ and $a_4 = -5$.

iii) $\mathbf{a = -1, d = 1/2}$

Ans: We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n - 1)d$ .....(1)

Substituting $a = -1, d = 1/2$ in (1) we get, $a_n = -1 + \frac{1}{2}(n - 1) = \frac{n - 3}{2}$ .....(2)

Therefore, from (2)

$a_1 = -1$, $a_2 = -\frac{1}{2}$, $a_3 = 0$ and $a_4 = \frac{1}{2}$.

iv) $\mathbf{a = -1.25, d = -0.25}$

Ans: We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n - 1)d$ .....(1)

Substituting $a=-1.25, d=-0.25$ in (1) we get,

$a_n=-1.25-0.25(n-1)=-1-0.25n$ .....(2)

Therefore, from (2)

$a_1=-1.25$, $a_2=-1.5$, $a_3=-1.75$ and $a_4=-2$.

3. For the following A.P.s, write the first term and the common difference.

i) $3,1,-1,-3,...$

Ans: From the given AP, we can see that the first term is 3.

The common difference is the difference between any two consecutive numbers of the A.P.

Common difference $=$ 2^{nd} term -1^{st} term

$\therefore$ Common difference $=$ $1-3=-2$.

ii) $-5,-1,3,7,...$

Ans: From the given AP, we can see that the first term is -5.

The common difference is the difference between any two consecutive numbers of the A.P.

Common difference $=$ 2^{nd} term -1^{st} term

$\therefore$ Common difference $=$ $-1-(-5)=4$.

iii) $\frac{1}{3},\frac{5}{3},\frac{9}{3},\frac{13}{3},...$

Ans: From the given AP, we can see that the first term is $\frac{1}{3}$.

The common difference is the difference between any two consecutive numbers of the A.P.

Common difference $=$ 2^{nd} term -1^{st} term

$\therefore$ Common difference $=$ $\frac{5}{3}-\frac{1}{3}=\frac{4}{3}$.

iv) $0.6,1.7,2.8,3.9,...$

Ans: From the given AP, we can see that the first term is 0.6.

The common difference is the difference between any two consecutive numbers of the A.P.

Common difference $=$ 2^{nd} term -1^{st} term

$\therefore$ Common difference $= 1.7 - 0.6 = 1.1$.

4. Which of the following are AP's? If they form an AP, find the common difference d and write three more terms.

i) 2,4,8,16...

Ans: For the given series, let us check the difference between all consecutive terms and find if they are equal or not.

$a_2 - a_1 = 4 - 2 = 2$ .....(1)

$a_3 - a_2 = 8 - 4 = 4$ .....(2)

$a_4 - a_3 = 16 - 8 = 8$ .....(3)

From (1), (2), and (3) we can see that the difference between all consecutive terms is not equal.

Therefore, the given series does not form an A.P.

ii) $2, \frac{5}{2}, 3, \frac{7}{2}, ...$

Ans: For the given series, let us check the difference between all consecutive terms and find if they are equal or not.

$a_2 - a_1 = \frac{5}{2} - 2 = \frac{1}{2}$ .....(1)

$a_3 - a_2 = 3 - \frac{5}{2} = \frac{1}{2}$ .....(2)

$a_4 - a_3 = \frac{7}{2} - 3 = \frac{1}{2}$ .....(3)

From (1), (2), and (3) we can see that the difference between all consecutive terms is equal.

Therefore, the given series form an A.P. with first term 2 and common difference $\frac{1}{2}$.

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ .....(4)

Substituting $a = 2, d = \frac{1}{2}$ in (1) we get, $a_n = 2 + \frac{1}{2}(n-1) = \frac{n+3}{2}$ .....(5)

Therefore, from (5)

$a_5 = 4$, $a_6 = \frac{9}{2}$ and $a_7 = 5$.

iii) 1.2,3.2,5.2,7.2...

Ans: For the given series, let us check the difference between all consecutive terms and find if they are equal or not.

$a_2 - a_1 = 3.2 - 1.2 = 2$ …..(1)

$a_3 - a_2 = 5.2 - 3.2 = 2$ …..(2)

$a_4 - a_3 = 7.2 - 5.2 = 2$ …..(3)

From (1), (2), and (3) we can see that the difference between all consecutive terms is equal.

Therefore, the given series form an A.P. with first term 1.2 and common difference 2.

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ …..(4)

Substituting $a = 1.2, d = 2$ in (1) we get, $a_n = 1.2 + 2(n-1) = 2n - 0.8$ …..(5)

Therefore, from (5)

$a_5 = 9.2$, $a_6 = 11.2$ and $a_7 = 13.2$.

iv) $-10,-6,-2,2,...$

Ans: For the given series, let us check the difference between all consecutive terms and find if they are equal or not.

$a_2 - a_1 = -6 - (-10) = 4$ …..(1)

$a_3 - a_2 = -2 - (-6) = 4$ …..(2)

$a_4 - a_3 = 2 - (-2) = 4$ …..(3)

From (1), (2), and (3) we can see that the difference between all consecutive terms is equal.

Therefore, the given series form an A.P. with first term -10 and common difference 4.

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ …..(4)

Substituting $a = -10, d = 4$ in (1) we get, $a_n = -10 + 4(n-1) = 4n - 14$ …..(5)

Therefore, from (5)

$a_5 = 6$, $a_6 = 10$ and $a_7 = 14$.

v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$

Ans: For the given series, let us check the difference between all consecutive terms and find if they are equal or not.

$a_2 - a_1 = \left(3+\sqrt{2}\right) - (3) = \sqrt{2}$ .....(1)

$a_3 - a_2 = \left(3+2\sqrt{2}\right) - \left(3+\sqrt{2}\right) = \sqrt{2}$ .....(2)

$a_4 - a_3 = \left(3+3\sqrt{2}\right) - \left(3+2\sqrt{2}\right) = \sqrt{2}$ .....(3)

From (1), (2), and (3) we can see that the difference between all consecutive terms is equal.

Therefore, the given series form an A.P. with first term 3 and common difference $\sqrt{2}$.

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ .....(4)

Substituting $a = 3, d = \sqrt{2}$ in (1) we get, $a_n = 3 + (n-1)\sqrt{2}$ .....(5)

Therefore, from (5)

$a_5 = 3 + 4\sqrt{2}$, $a_6 = 3 + 5\sqrt{2}$ and $a_7 = 3 + 6\sqrt{2}$.

vi) 0.2,0.22,0.222,0.2222.....

Ans: For the given series, let us check the difference between all consecutive terms and find if they are equal or not.

$a_2 - a_1 = 0.22 - 0.2 = 0.02$ .....(1)

$a_3 - a_2 = 0.222 - 0.22 = 0.002$ .....(2)

$a_4 - a_3 = 0.2222 - 0.222 = 0.0002$ .....(3)

From (1), (2), and (3) we can see that the difference between all consecutive terms is not equal.

Therefore, the given series does not form an A.P.

vii) 0,–4,–8,–12....

Ans: For the given series, let us check the difference between all consecutive terms and find if they are equal or not.

$a_2 - a_1 = -4 - 0 = -4$ .....(1)

$a_3 - a_2 = -8 - (-4) = -4$ .....(2)

$a_4 - a_3 = -12 - (-8) = -4$ .....(3)

From (1), (2), and (3) we can see that the difference between all consecutive terms is equal.

Therefore, the given series form an A.P. with first term 0 and common difference -4.

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ .....(4)

Substituting $a = 0, d = -4$ in (1) we get, $a_n = 0 - 4(n-1) = 4 - 4n$ .....(5)

Therefore, from (5)

$a_5 = -16$, $a_6 = -20$ and $a_7 = -24$.

viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}....$

Ans: For the given series, let us check the difference between all consecutive terms and find if they are equal or not.

$a_2 - a_1 = \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right) = 0$ .....(1)

$a_3 - a_2 = \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right) = 0$ .....(2)

$a_4 - a_3 = \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right) = 0$ .....(3)

From (1), (2), and (3) we can see that the difference between all consecutive terms is equal.

Therefore, the given series form an A.P. with first term $-\frac{1}{2}$ and common difference 0.

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ .....(4)

Substituting $a = -\frac{1}{2}, d = 0$ in (1) we get, $a_n = -\frac{1}{2} + 0(n-1) = -\frac{1}{2}$ .....(5)

Therefore, from (5)

$a_5 = -\frac{1}{2}$, $a_6 = -\frac{1}{2}$ and $a_7 = -\frac{1}{2}$.

Exercise 5.2

1. Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the n^{th} term of the A.P.

	a	d	n	a_n
I	7	3	8	
II	–18		10	0
III		–3	18	–5
IV	–18.9	2.5		3.6
V	3.5	0	105	

I. **Ans:** Given, the first Term, $a = 7$ ….. (1)

Given, the common Difference, $d = 3$ …..(2)

Given, the number of Terms, $n = 8$ …..(3)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ …..(4)

Substituting the values from (1), (2) and (3) in (4) we get,

$a_n = 7 + (8-1)3$

$\Rightarrow a_n = 7 + 21$

$\therefore a_n = 28$

II. **Ans:** Given, the first Term, $a = -18$ ….. (1)

Given, the n^{th} term, $a_n = 0$ …..(2)

Given, the number of Terms, $n = 10$ …..(3)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ …..(4)

Substituting the values from (1), (2) and (3) in (4) we get,

$0 = -18 + (10-1)d$

$\Rightarrow 18 = 9d$

$\therefore d = 2$

III. **Ans:** Given, the n^{th} term, $a_n = -5$ ….. (1)

Given, the common Difference, $d = -3$ …..(2)

Given, the number of Terms, $n = 18$ …..(3)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ …..(4)
Substituting the values from (1), (2) and (3) in (4) we get,
$-5 = a + (18-1)(-3)$
$\Rightarrow -5 = a - 51$
$\therefore a_n = 46$

IV. **Ans:** Given, the first Term, $a = -18.9$ ….. (1)
Given, the common Difference, $d = 2.5$ …..(2)
Given, the n^{th} term, $a_n = 3.6$ …..(3)
We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ …..(4)
Substituting the values from (1), (2) and (3) in (4) we get,
$3.6 = -18.9 + (n-1)(2.5)$
$\Rightarrow 22.5 = (n-1)(2.5)$
$\Rightarrow 9 = (n-1)$
$\therefore a_n = 10$

V. **Ans:** Given, the first Term, $a = 3.5$ ….. (1)
Given, the common Difference, $d = 0$ …..(2)
Given, the number of Terms, $n = 105$ …..(3)
We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ …..(4)
Substituting the values from (1), (2) and (3) in (4) we get,
$a_n = 3.5 + (105-1)(0)$
$\therefore a_n = 3.5$

2. Choose the correct choice in the following and justify

I. 30^{th} term of the A.P $10, 7, 4, \ldots,$ is
A. 97
B. 77
C. –77
D. –87
Ans: C. -77
Given, the first Term, $a = 10$ ….. (1)
Given, the common Difference, $d = 7 - 10 = -3$ …..(2)
Given, the number of Terms, $n = 30$ …..(3)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ …..(4)

Substituting the values from (1), (2) and (3) in (4) we get, $a_n = 10 + (30-1)(-3)$

$\Rightarrow a_n = 10 - 87$

$\therefore a_n = -77$

II. 11^{th} term of the A.P $-3, -\frac{1}{2}, 2, \ldots,$ is

i) 28

ii) 22

iii) –38

iv) $-48\frac{1}{2}$

Ans: B. 22

Given, the first Term, $a = -3$ ….. (1)

Given, the common Difference, $d = -\frac{1}{2} - (-3) = \frac{5}{2}$ …..(2)

Given, the number of Terms, $n = 11$ …..(3)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ …..(4)

Substituting the values from (1), (2) and (3) in (4) we get, $a_n = -3 + \frac{5}{2}(11-1)$

$\Rightarrow a_n = -3 + 25$

$\therefore a_n = 22$

3. In the following APs find the missing term in the blanks

I. 2,__,26

Ans: Given, first term $a = 2$ …..(1)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$

Substituting the values from (1) we get, $a_n = 2 + (n-1)d$ …..(2)

Given, third term $a_3 = 26$. From (2) we get,

$26 = 2 + (3-1)d$

$\Rightarrow 26 = 2 + 2d$

$\therefore d = 12$ ….(3)

From (1), (2) and (3) we get for $n = 2$

$a_2 = 2 + (2 - 1)(12)$

$\therefore a_2 = 14$

$\therefore$ The sequence is $2, 14, 26$.

II.___,13,__,3

Ans: Given, second term $a_2 = 13$ …..(1)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n - 1)d$ …..(2)

Substituting the values from (1) for $n = 2$ we get, $13 = a + d$ …..(3)

Given, fourth term $a_4 = 3$. From (2) we get,

$3 = a + 3d$ …..(4)

Solving (3) and (4) by subtracting (3) from (4) we get,

$3 - 13 = (a + 3d) - (a + d)$

$\Rightarrow -10 = 2d$

$\therefore d = -5$ ….(5)

From (3) and (5) we get

$13 = a - 5$

$\Rightarrow a = 18$ ……(6)

Substituting the values from (5) and (6) in (2) we get,

$a_n = 18 - 5(n - 1)$ …..(7)

First term, $a = 18$ and third term $a_3 = 8$

$\therefore$ The sequence is $18, 13, 8, 3$.

III. 5,__,__,$9\frac{1}{2}$

Ans: Given, first term $a = 5$ …..(1)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n - 1)d$ …..(2)

Substituting the values from (1) in (2) we get, $a_n = 5 + (n - 1)d$ …..(3)

Given, fourth term $a_4 = 9\frac{1}{2}$. From (3) we get,

$9\frac{1}{2} = 5 + (4 - 1)d$

$\Rightarrow 9\frac{1}{2} = 5 + 3d$

$\therefore d = \frac{3}{2}$ ....(4)

From (3) and (4) we get

$a_n = 5 + \frac{3}{2}(n-1)$ ......(5)

Second term, $a_2 = \frac{13}{2}$ and third term $a_3 = 8$

$\therefore$ The sequence is $5, \frac{13}{2}, 8, 9\frac{1}{2}$.

IV. -4,__,__,__,__,6

Ans: Given, first term $a = -4$ .....(1)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ .....(2)

Substituting the values from (1) in (2) we get, $a_n = -4 + (n-1)d$ .....(3)

Given, sixth term $a_6 = 6$. From (3) we get,

$6 = -4 + (6-1)d$

$\Rightarrow 6 = -4 + 5d$

$\therefore d = 2$ ....(4)

From (3) and (4) we get

$a_n = -4 + 2(n-1)$ ......(5)

Second term $a_2 = -2$, third term $a_3 = 0$, fourth term $a_4 = 2$ and fifth term $a_5 = 4$.

$\therefore$ The sequence is $-4, -2, 0, 2, 4, 6$

V.___,38,__,__,__,-22

Ans: Given, second term $a_2 = 38$ .....(1)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ .....(2)

Substituting the values from (1) for $n = 2$ we get, $38 = a + d$ .....(3)

Given, sixth term $a_6 = -22$. From (2) we get,

$-22 = a + 5d$ .....(4)

Solving (3) and (4) by subtracting (3) from (4) we get,

$-22-38=(a+5d)-(a+d)$

$\Rightarrow -60=4d$

$\therefore d=-15$ ….(5)

From (3) and (5) we get

$38=a-15$

$\Rightarrow a=53$ ……(6)

Substituting the values from (5) and (6) in (2) we get,

$a_n=53-15(n-1)$ …..(7)

First term, $a=53$, second term $a_3=23$, third term $a_3=8$ and fourth term $a_4=-7$

$\therefore$ The sequence is $53,38,23,8,–7,–22$.

4. Which term of the A.P. $3,8,13,18,...$ is 78?

Ans: Given, the first Term, $a=3$ ….. (1)

Given, the common Difference, $d=8-3=5$ …..(2)

Given, the n^{th} term, $a_n=78$ …..(3)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n=a+(n-1)d$ …..(4)

Substituting the values from (1), (2) and (3) in (4) we get,

$78=3+5(n-1)$

$\Rightarrow 75=5(n-1)$

$\Rightarrow 15=(n-1)$

$\therefore n=16$

Therefore, 16^{th} term of this A.P. is 78.

5. Find the number of terms in each of the following A.P.

I. 7,13,19,...,205

Ans: Given, the first Term, $a=7$ ….. (1)

Given, the common Difference, $d=13-7=6$ …..(2)

Given, the n^{th} term, $a_n=205$ …..(3)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n=a+(n-1)d$ …..(4)

Substituting the values from (1), (2) and (3) in (4) we get,

$205=7+6(n-1)$

$\Rightarrow 198 = 6(n - 1)$

$\Rightarrow 33 = (n - 1)$

$\therefore n = 34$

Therefore, given A.P. series has 34 terms.

II. $18, 15\frac{1}{2}, 13, \ldots, -47$

Ans: Given, the first Term, $a = 18$ ….. (1)

Given, the common Difference, $d = 15\frac{1}{2} - 18 = -\frac{5}{2}$ …..(2)

Given, the n^{th} term, $a_n = -47$ …..(3)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n - 1)d$ …..(4)

Substituting the values from (1), (2) and (3) in (4) we get,

$-47 = 18 - \frac{5}{2}(n - 1)$

$\Rightarrow -65 = -\frac{5}{2}(n - 1)$

$\Rightarrow 26 = (n - 1)$

$\therefore n = 27$

Therefore, given A.P. series has 27 terms.

6. Check whether -150 is a term of the A.P. $11, 8, 5, 2, \ldots$

Ans: Given, the first Term, $a = 11$ ….. (1)

Given, the common Difference, $d = 8 - 11 = -3$ …..(2)

Given, the n^{th} term, $a_n = -150$ …..(3)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n - 1)d$ …..(4)

Substituting the values from (1), (2) and (3) in (4) we get,

$-150 = 11 - 3(n - 1)$

$\Rightarrow -161 = -3(n - 1)$

$\Rightarrow \frac{161}{3} = (n - 1)$

$\therefore n = \frac{164}{3}$

Since n is nor a natural number. Therefore, -150 is not a term of the given A.P. series.

7. Find the 31^{st} term of an A.P. whose 11^{th} term is 38 and the 16^{th} term is 73

Ans: Given, the 11^{th} Term, $a_{11} = 38$ ..... (1)

Given, the 16^{th} Term, $a_{16} = 73$ .....(2)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ .....(3)

Substituting the values from (1) in (3) we get,

$38 = a + (11-1)d$

$\Rightarrow 38 = a + 10d$ .....(4)

Substituting the values from (2) in (3) we get,

$73 = a + (16-1)d$

$\Rightarrow 73 = a + 15d$ .....(5)

Solving equations (4) and (5) by subtracting (4) from (5) we get,

$\Rightarrow 73 - 38 = (a + 15d) - (a + 10d)$

$\Rightarrow 5 = 35d$

$\therefore d = 7$ .....(6)

Substituting value from (6) in (4) we get,

$\Rightarrow 38 = a + 70$

$\therefore a = -32$ .....(7)

Again, substituting the values from (6) and (7) in (3) we get,

$a_n = -32 + 7(n-1)$ .....(8)

To find the 31^{st} term substitute $n = 31$ in (8) we get,

$a_{31} = -32 + 7(31-1)$

$\Rightarrow a_{31} = -32 + 210$

$\therefore a_{31} = 178$

Therefore, the 31^{st} term of an A.P. whose 11^{th} term is 38 and the 16^{th} term is 73 is 178.

8. An A.P. consists of 50 terms of which 3^{rd} term is 12 and the last term is 106. Find the 29^{th} term.

Ans: Given, the 3^{rd} Term, $a_3 = 12$ ..... (1)

Given, the 50^{th} Term, $a_{50} = 106$ .....(2)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ .....(3)

Substituting the values from (1) in (3) we get,

$12 = a + (3-1)d$

$\Rightarrow 12 = a + 2d$ .....(4)

Substituting the values from (2) in (3) we get,

$106 = a + (50-1)d$

$\Rightarrow 106 = a + 49d$ .....(5)

Solving equations (4) and (5) by subtracting (4) from (5) we get,

$\Rightarrow 106 - 12 = (a + 49d) - (a + 2d)$

$\Rightarrow 94 = 47d$

$\therefore d = 2$ .....(6)

Substituting value from (6) in (4) we get,

$\Rightarrow 12 = a + 4$

$\therefore a = 8$ .....(7)

Again, substituting the values from (6) and (7) in (3) we get,

$a_n = 8 + 2(n-1)$ .....(8)

To find the 29^{th} term substitute $n = 29$ in (8) we get,

$a_{29} = 8 + 2(29-1)$

$\Rightarrow a_{29} = 8 + 56$

$\therefore a_{29} = 64$

Therefore, the 29^{th} term of the A.P. is 64.

9. If the 3^{rd} and the 9^{th} terms of an A.P. are 4 and -8 respectively. Which term of this A.P. is zero.

Ans: Given, the 3^{rd} Term, $a_3 = 4$ ..... (1)

Given, the 9^{th} Term, $a_9 = -8$ .....(2)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ .....(3)

Substituting the values from (1) in (3) we get,

$4 = a + (3-1)d$

$\Rightarrow 4 = a + 2d$ .....(4)

Substituting the values from (2) in (3) we get,

$-8 = a + (9-1)d$

$\Rightarrow -8 = a + 8d \quad .....(5)$
Solving equations (4) and (5) by subtracting (4) from (5) we get,
$\Rightarrow -8 - 4 = (a + 8d) - (a + 2d)$
$\Rightarrow -12 = 6d$
$\therefore d = -2 \quad .....(6)$
Substituting value from (6) in (4) we get,
$\Rightarrow 4 = a - 4$
$\therefore a = 8 \quad .....(7)$
Again, substituting the values from (6) and (7) in (3) we get,
$a_n = 8 - 2(n - 1) \quad .....(8)$
T find the term which is zero, substitute $a_n = 0$ in (8)
$0 = 8 - 2(n - 1)$
$\Rightarrow 8 = 2(n - 1)$
$\Rightarrow 4 = (n - 1)$
$\therefore n = 5$
Therefore, given A.P. series has 5^{th} term as zero.

10. If 17^{th} term of an A.P. exceeds its 10^{th} term by 7. Find the common difference.

Ans: Given that the 17^{th} term of an A.P. exceeds its 10^{th} term by 7 i.e.,
$a_{17} = a_{10} + 7 \quad .....(1)$
We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n - 1)d \quad .....(2)$
For 17^{th} term substitute $n = 17$ in (2) i.e., $a_{17} = a + 16d \quad .....(3)$
For 10^{th} term substitute $n = 10$ in (2) i.e., $a_{10} = a + 9d \quad .....(4)$
Therefore, from (1), (3) and (4) we get,
$a + 16d = a + 9d + 7$
$\Rightarrow 7d = 7$
$\therefore d = 1$
Therefore, the common difference is 1.

11. Which term of the A.P. $3, 15, 27, 39, ...$ will be 132 more than its 54^{th} term?

Ans: Let n^{th} term of A.P. be 132 more than its 54^{th} term i.e.,

$a_n = a_{54} + 132$ .....(1)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ ..... (2)

For 54^{th} term substitute $n = 54$ in (2) i.e., $a_{54} = a + 53d$ ..... (3)

Therefore, from (1), (2) and (3) we get,

$a + (n-1)d = a + 53d + 132$

$\Rightarrow (n-1)d - 53d = 132$

$\therefore d = \dfrac{132}{n-54}$ ..... (4)

Now, given A.P. $3, 15, 27, 39, ...$

Common difference $d = 15 - 3 = 12$ ..... (5)

Hence, from (4) and (5) we get $12 = \dfrac{132}{n-54}$

$\Rightarrow n - 54 = 11$

$\therefore n = 65$

Therefore, 65^{th} term of the given A.P. will be 132 more than its 54^{th} term.

12. Two APs have the same common difference. The difference between their 100^{th} term is 100, what is the difference between their 1000^{th} terms?

Ans: Let 2 A.P.'s be

$a, a+d, a+2d, a+3d, ...$ .....(1)

$b, b+d, b+2d, b+3d, ...$ .....(2)

(Since common difference is same)

Given that the difference between their 100^{th} term is 100 i.e.,

$a_{100} - b_{100} = 100$ .....(3)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ ..... (4)

Therefore, from (3) and (4) we get,

$a + (100-1)d - (b + (100-1)d) = 100$

$\Rightarrow a - b = 100$ ..... (5)

Similarly, the difference between their 1000^{th} terms is,

$a_{1000} - b_{1000} = [a + (1000-a)d] - [b + (1000-a)d]$

$\Rightarrow a_{1000} - b_{1000} = a - b$

$\therefore a_{1000} - b_{1000} = 100$

Therefore, the difference between their 1000^{th} terms is 100.

13. How many three-digit numbers are divisible by 7?

Ans: First three-digit number that is divisible by 7 is 105 then the next number will be $105+7=112$.

Therefore, the series becomes $105,112,119,....$

This is an A.P. having first term as 105 and common difference as 7.

Now, the largest 3 digit number is 999.

Leu us divide it by 7, to get the remainder.

$999=142\times 7+5$

Therefore, $999-5=994$ is the maximum possible three-digit number that is divisible by 7.

Also, this will be the last term of the A.P. series.

Hence the final series is as follows: $105,112,119,...,994$

Let 994 be the n^{th} term of this A.P.

Then, $a_n=105+7(n-1)$

$\Rightarrow 994=105+7(n-1)$

$\Rightarrow 889=7(n-1)$

$\Rightarrow 127=(n-1)$

$\therefore n=128$

Therefore, 128 three-digit numbers are divisible by 7.

14. How many multiples of 4 lie between 10 and 250?

Ans: First number that is divisible by 4 and lie between 10 and 250 is 12. The next number will be $12+4=16$.

Therefore, the series becomes $12,16,20,....$

This is an A.P. having first term as 12 and common difference as 4.

Now, the largest number in range is 250.

Leu us divide it by 4 to get the remainder.

$250=62\times 4+2$

Therefore, $250-2=248$ is the last term of the A.P. series.

Hence the final series is as follows: $12,16,20,....,248$

Let 248 be the n^{th} term of this A.P.

Then, $a_n=12+4(n-1)$

$\Rightarrow 248=12+4(n-1)$

$\Rightarrow 236 = 4(n-1)$

$\Rightarrow 59 = (n-1)$

$\therefore n = 60$

Therefore, 60 multiples of 4 lie between 10 and 250.

15. For what value of n, are the n^{th} terms of two APs $63, 65, 67, ...$ and $3, 10, 17, ...$ equal

Ans: Given 2 A.P.'s are

$63, 65, 67, ...(1)$

Its first term is 63 and common difference is $65-63=2$

$3, 10, 17, ... (2)$

Its first term is 3 and common difference is $10-3=7$

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a+(n-1)d$ ..... (3)

Therefore, from (1) and (3) we get the n^{th} term of the first A.P. is

$a_n = 63+2(n-1)$

$\Rightarrow a_n = 61+2n$ ..... (4)

And from (2) and (3) we get the n^{th} term of the second A.P. is

$b_n = 3+7(n-1)$

$\Rightarrow b_n = -4+7n$ ..... (5)

If the n^{th} terms of two APs $63, 65, 67, ...$ and $3, 10, 17, ...$ are equal the from (4) and (5),

$a_n = b_n$

$\Rightarrow 61+2n = -4+7n$

$\Rightarrow 65 = 5n$

$\therefore n = 13$

Therefore, the 13^{th} term of both the A.P.'s are equal.

16. Determine the A.P. whose third term is 16 and the 7^{th} term exceeds the 5^{th} term by 12.

Ans: Given the 7^{th} term of A.P. is 12 more than its 5^{th} term i.e.,

$a_7 = a_5 + 12$ .....(1)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a+(n-1)d$ ..... (2)

For 5^{th} term substitute $n=5$ in (2) i.e., $a_5 = a+4d$ ….. (3)

For 7^{th} term substitute $n=7$ in (2) i.e., $a_7 = a+6d$ ….. (4)

Therefore, from (1), (3) and (4) we get,

$a+6d = a+4d+12$

$\Rightarrow 2d = 12$

$\therefore d = 6$ ….. (5)

Substituting (5) in (2) we get, $a_n = a+6(n-1)$ ……(6)

Given the third term of the A.P. is 16. Hence from (6),

$16 = a+6(3-1)$

$\Rightarrow 16 = a+12$

$\therefore a = 4$ ….. (7)

Hence from (6), $a_n = 4+6(n-1)$

Therefore, the A.P. will be $4,10,16,22,.....$

17. Find the 20^{th} term from the last term of the A.P. $3,8,13,...,253$

Ans: Given A.P. $3,8,13,...,253$. To find the 20^{th} term from the last write the given A.P. in reverse order and then find its 20^{th} term.

Required A.P. is $253,...,13,8,3$ ….. (1)

Its first A.P. is 253 and common difference is $8-13=-5$ ....(2)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a+(n-1)d$ ….. (3)

Hence from (2) and (3) we get, $a_n = 253-5(n-1)$ …..(4)

Substitute $n=20$ in (4) we get,

$a_{20} = 253-5(20-1)$

$\Rightarrow a_{20} = 158.$

Therefore, 20^{th} term from the last is 158.

18. The sum of 4^{th} and 8^{th} terms of an A.P. is 24 and the sum of the 6^{th} and 10^{th} terms is 44. Find the first three terms of the A.P.

Ans: Given the sum of 4^{th} and 8^{th} terms of an A.P. is 24 i.e.,

$a_4 + a_8 = 24$ …..(1)

Given the sum of 6^{th} and 10^{th} terms is 44 i.e.,

$a_6 + a_{10} = 44$ …..(2)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ ….. (3)

For 4^{th} term substitute $n = 4$ in (3) i.e., $a_4 = a + 3d$ ….. (4)

For 6^{th} term substitute $n = 6$ in (3) i.e., $a_6 = a + 5d$ ….. (5)

For 8^{th} term substitute $n = 8$ in (3) i.e., $a_8 = a + 7d$ ….. (6)

For 10^{th} term substitute $n = 10$ in (3) i.e., $a_{10} = a + 9d$ ….. (7)

Therefore, from (1), (6) and (4) we get,

$(a + 3d) + (a + 7d) = 24$

$\Rightarrow 2a + 10d = 24$

$\Rightarrow a + 5d = 12$ ….. (8)

From (2), (5) and (7) we get,

$(a + 5d) + (a + 9d) = 44$

$\Rightarrow 2a + 14d = 44$

$\Rightarrow a + 7d = 22$ ….. (9)

Subtracting (8) from (9) we get,

$\Rightarrow (a + 7d) - (a + 5d) = 22 - 12$

$\Rightarrow 2d = 10$

$\therefore d = 5$ ….. (10)

Substituting this value from (10) in (9) we get,

$a + 35 = 22$

$\therefore a = -13$ ….. (11)

Therefore from (10) and (11), the first three terms of the A.P. are $-13, -8, -3$.

19. Subba Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

Ans: Given in the first year, annual salary is Rs 5000.

In the second year, annual salary is Rs $5000 + 200 = 5200$.

In the third year, annual salary is Rs $5200 + 200 = 5400$.

This series will form an A.P. with first term 5000 and common difference 200.

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$

Therefore, In the n^{th} year, annual salary is $a_n = 5000 + 200(n-1)$

$\Rightarrow a_n = 4800 + 200n$ …. (1)

To find the year in which his annual income reaches Rs 7000, substitute $a_n = 7000$ in (1) and find the value of n i.e.,

$7000 = 4800 + 200n$

$\Rightarrow 2200 + 200n$

$\therefore n = 11$

Therefore, in 11^{th} year i.e., in 2005 his salary will be Rs 7000.

20. Ramkali saved Rs 5 in the first week of a year and then increased her weekly saving by Rs 1.75. If in the n^{th} week, her weekly savings become Rs 20.75, find n.

Ans: Given in the first week the savings is Rs 5.

In the second week the savings is Rs $5 + 1.75 = 6.75$.

In the third week the savings is Rs $6.75 + 1.75 = 8.5$.

This series will form an A.P. with first term 5 and common difference 1.75.

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$

Therefore, In the n^{th} week the savings is $a_n = 5 + 1.75(n-1)$

$\Rightarrow a_n = 3.25 + 1.75n$ …. (1)

To find the week in which her savings reaches Rs 20.75, substitute $a_n = 20.75$ in (1) and find the value of n i.e.,

$20.75 = 3.25 + 1.75n$

$\Rightarrow 17.5 = 1.75n$

$\therefore n = 10$

Therefore, in 10^{th} week her savings will be Rs 20.75.

Exercise 5.3

1. Find the sum of the following APs.

i) 2,7,12,.... to 10 terms.

Ans: Given, the first Term, $a = 2$ ….. (1)

Given, the common Difference, $d = 7 - 2 = 5$ …..(2)

Given, the number of Terms, $n = 10$ …..(3)

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ …..(4)

Substituting the values from (1), (2) and (3) in (4) we get,

$$S_n = \frac{10}{2}\left[2(2) + (10-1)(5)\right]$$

$\Rightarrow S_n = 5[4+45]$

$\therefore S_n = 245$

ii) $-37, -33, -29, \ldots$ to 12 terms

Ans: Given, the first Term, $a = -37$ ….. (1)

Given, the common Difference, $d = -33-(-37) = 4$ …..(2)

Given, the number of Terms, $n = 12$ …..(3)

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}[2a+(n-1)d]$ …..(4)

Substituting the values from (1), (2) and (3) in (4) we get, $S_n = \frac{12}{2}[2(-37)+(12-1)(4)]$

$\Rightarrow S_n = 6[-74+44]$

$\therefore S_n = -180$

iii) $0.6, 1.7, 2.8, \ldots\ldots$ to 100 terms

Ans: Given, the first Term, $a = 0.6$ ….. (1)

Given, the common Difference, $d = 1.7-0.6 = 1.1$ …..(2)

Given, the number of Terms, $n = 100$ …..(3)

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}[2a+(n-1)d]$ …..(4)

Substituting the values from (1), (2) and (3) in (4) we get, $S_n = \frac{100}{2}[2(0.6)+(100-1)(1.1)]$

$\Rightarrow S_n = 50[1.2+108.9]$

$\therefore S_n = 5505$

iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots\ldots$ to 11 terms

Ans: Given, the first Term, $a = \frac{1}{15}$ ….. (1)

Given, the common Difference, $d=\frac{1}{12}-\frac{1}{15}=\frac{1}{60}$ …..(2)

Given, the number of Terms, $n=11$ …..(3)

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$ …..(4)

Substituting the values from (1), (2) and (3) in (4) we get,

$$S_n=\frac{11}{2}\left[2\left(\frac{1}{15}\right)+(11-1)\left(\frac{1}{60}\right)\right]$$

$$\Rightarrow S_n=\frac{11}{2}\left[\frac{4+5}{30}\right]$$

$$\therefore S_n=\frac{33}{20}$$

2. Find the sums given below

i) $7+10\frac{1}{2}+14+......+84$

Ans: Given, the first Term, $a=7$ ….. (1)

Given, the common Difference, $d=10\frac{1}{2}-7=\frac{7}{2}$ …..(2)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n=a+(n-1)d$ ….. (3)

Substituting the values from (1) and (2) in (3) we get,

$a_n=7+\frac{7}{2}(n-1)=\frac{7}{2}(n+1)$ ….. (4)

Given, last term of the series, $a_n=84$ …..(5)

Substituting (5) in (4) we get, $84=\frac{7}{2}(n+1)$

$$\Rightarrow 24=(n+1)$$

$\therefore n=23$ ……(6)

We know that the sum of n terms of the A.P. with first term a and last term l is given by $S_n=\frac{n}{2}[a+l]$ …..(7)

Substituting the values from (1), (5) and (6) in (7) we get, $S_n=\frac{23}{2}[7+84]$

$\Rightarrow S_n = \frac{23}{2}(91)$

$\therefore S_n = 1046\frac{1}{2}$

ii) $34+32+30+.....+10$

Ans: Given, the first Term, $a = 34$....... (1)

Given, the common Difference, $d = 32 - 34 = -2$ .....(2)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ ..... (3)

Substituting the values from (1) and (2) in (3) we get,

$a_n = 34 - 2(n-1) = 36 - 2n$ ..... (4)

Given, last term of the series, $a_n = 10$ .....(5)

Substituting (5) in (4) we get, $10 = 36 - 2n$

$\Rightarrow 2n = 26$

$\therefore n = 13$ ......(6)

We know that the sum of n terms of the A.P. with first term a and last term l is given by $S_n = \frac{n}{2}[a + l]$ .....(7)

Substituting the values from (1), (5) and (6) in (7) we get, $S_n = \frac{13}{2}[34 + 10]$

$\Rightarrow S_n = \frac{13}{2}(44)$

$\therefore S_n = 286$

iii) $-5 + (-8) + (-11) + ... + (-230)$

Ans: Given, the first Term, $a = -5$ ..... (1)

Given, the common Difference, $d = -8 - (-5) = -3$ .....(2)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ ..... (3)

Substituting the values from (1) and (2) in (3) we get,

$a_n = -5 - 3(n-1) = -2 - 3n$ ..... (4)

Given, last term of the series, $a_n = -230$ .....(5)

Substituting (5) in (4) we get, $-230 = -2 - 3n$

$\Rightarrow -228 = -3n$

$\therefore n = 76$ ......(6)

We know that the sum of n terms of the A.P. with first term a and last term l is given by $S_n = \frac{n}{2}[a + l]$ …..(7)

Substituting the values from (1), (5) and (6) in (7) we get, $S_n = \frac{76}{2}\left[-5 + (-230)\right]$

$\Rightarrow S_n = \frac{76}{2}(-235)$

$\therefore S_n = -8930$

3. In an AP

i) Given $a = 5,\ d = 3,\ a_n = 50$, find n and S_n.

Ans: Given, the first Term, $a = 5$ ….. (1)

Given, the common Difference, $d = 3$ …..(2)

Given, n^{th} term of the A.P., $a_n = 50$ …..(3)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ ….. (4)

Substituting the values from (1), (2) and (3) in (4) we get,

$50 = 5 + 3(n-1) = 2 + 3n$

Simplifying it further we get,

$n = \frac{50-2}{3}$

$\therefore n = 16$ …..(5)

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ …..(6)

Substituting the values from (1), (2) and (5) in (6) we get,

$S_n = \frac{16}{2}\left[2(5) + (16-1)(3)\right]$

$\Rightarrow S_n = 8[10 + 45]$

$\therefore S_n = 440$

ii) Given $a = 7,\ a_{13} = 35$, find d and S_{13}.

Ans: Given, the first Term, $a = 7$ ….. (1)

Given, 13^{th} term of the A.P., $a_{13} = 35$ …..(2)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ ….. (3)

Substituting the values from (1), (2) in (3) we get,

$35 = 7 + (13-1)d = 7 + 12d$

Simplifying it further we get,

$d = \frac{28}{12}$

$\therefore d = \frac{7}{3}$ …..(4)

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}[2a + (n-1)d]$ …..(5)

Substituting the values from (1) and (4) in (5) we get,

$S_{13} = \frac{13}{2}\left[2(7) + (13-1)\left(\frac{7}{3}\right)\right]$

$\Rightarrow S_{13} = \frac{13}{2}[14 + 28]$

$\therefore S_{13} = 273$

iii) Given $d = 3$, $a_{12} = 37$, find a and S_{12}.

Ans: Given, the common difference, $d = 3$ ….. (1)

Given, 12^{th} term of the A.P., $a_{12} = 37$ …..(2)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ ….. (3)

Substituting the values from (1), (2) in (3) we get,

$37 = a + 3(12-1) = a + 33$

Simplifying it further we get,

$\therefore a = 4$ …..(4)

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}[2a + (n-1)d]$ …..(5)

Substituting the values from (1) and (4) in (5) we get,

$S_{12} = \frac{12}{2}\left[2(4) + (12-1)(3)\right]$

$\Rightarrow S_{12} = 6[8 + 33]$

$\therefore S_{12} = 246$

iv) Given $a_3 = 15$, $S_{10} = 125$ find a_{10} and d.

Ans: Given, 3[rd] term of the A.P., $a_3 = 15$ …..(1)

Given, the sum of terms, $S_{10} = 125$ ….. (2)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ ….. (3)

Substituting the values from (1) in (3) we get,

$15 = a + (3-1)d = a + 2d$ …..(4)

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ …..(5)

Substituting the values from (1) in (5) we get, $125 = \frac{10}{2}\left[2a + (10-1)d\right]$

$\Rightarrow 125 = 5\left[2a + 9d\right]$

$\therefore 25 = 2a + 9d$ …..(5)

Let us solve equations (4) and (5) by subtracting twice of (4) from (5) we get,

$25 - 30 = (2a + 9d) - (2a + 4d)$

$\Rightarrow -5 = 5d$

$\therefore d = -1$ …..(6)

From (4) and (6) we get, $a = 17$ …..(7)

From (3), (6) and (7) for $n = 10$ we get,

$a_{10} = 17 - (10-1)$

$\therefore a_{10} = 8$

v) Given $S_9 = 75$, $d = 5$ find a and a_9.

Ans: Given, common difference, $d = 5$ …..(1)

Given, the sum of terms, $S_9 = 75$ ….. (2)

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ …..(3)

Substituting the values from (1), (2) in (3) we get, $75 = \frac{9}{2}\left[2a + 5(9-1)\right]$

$\Rightarrow 25 = 3\left[a + 20\right]$

$\Rightarrow 3a = -35$

$\therefore a = -\dfrac{35}{3}$ …..(4)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ ….. (5)

Substituting the values from (1), (4) in (5) we get,

$a_9 = -\dfrac{35}{3} + 5(9-1)$

$\Rightarrow a_9 = -\dfrac{35}{3} + 40$

$\therefore a_9 = \dfrac{85}{3}$

vi) Given $a = 2$, $d = 8$, $S_n = 90$, find n and a_n.

Ans: Given, common difference, $d = 8$ …..(1)

Given, first term, $a = 2$ …..(2)

Given, the sum of terms, $S_n = 90$ ….. (3)

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$ …..(3)

Substituting the values from (1), (2), (3) in (4) we get, $90 = \dfrac{n}{2}\left[2(2) + 8(n-1)\right]$

$\Rightarrow 45 = n[2n - 1]$

$\Rightarrow 2n^2 - n - 45 = 0$

$\Rightarrow 2n^2 - 10n + 9n - 45 = 0$

$\Rightarrow 2n(n-5) + 9(n-5) = 0$

$\Rightarrow (n-5)(2n+9) = 0$

$\therefore n = 5$ …..(4)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ ….. (5)

Substituting the values from (1), (2), (4) in (5) we get,

$a_5 = 2 + 8(5-1)$

$\Rightarrow a_5 = 2 + 32$

$\therefore a_5 = 34$

vii) Given $a = 8$, $S_n = 210$, $a_n = 62$, find n and d.

Ans: Given, first term, $a = 8$ …..(1)

Given, the sum of terms, $S_n = 210$ ….. (2)

Given, the n^{th} term, $a_n = 62$ …..(3)

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ …..(4)

Substituting the values from (1), (2) in (4) we get, $210 = \frac{n}{2}\left[2(8) + d(n-1)\right]$

$\Rightarrow 420 = n\left[16 + (n-1)d\right]$ …..(4)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ ….. (5)

Substituting the values from (1), (3) in (5) we get,

$62 = 8 + (n-1)d$ …..(6)

Let us solve equations (4) and (6) by subtracting n times of (6) from (4) we get,

$420 - 62n = \left(16n + n(n-1)d\right) - \left(8n + n(n-1)d\right)$

$\Rightarrow 420 - 62n = 8n$

$\Rightarrow 420 = 70n$

$\therefore n = 6$ ……(7)

Substituting the values from (7) in (6) we get,

$62 = 8 + (6-1)d$

$\Rightarrow 54 = 5d$

$\therefore d = \frac{54}{5}$

viii) Given $S_n = -14$, $d = 2$, $a_n = 4$, find n and a.

Ans: Given, common difference, $d = 2$ …..(1)

Given, the sum of terms, $S_n = -14$ ….. (2)

Given, the n^{th} term, $a_n = 4$ …..(3)

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ …..(4)

Substituting the values from (1), (2) in (4) we get, $-14 = \frac{n}{2}\left[2a+2(n-1)\right]$

$\Rightarrow -14 = n\left[a+n-1\right]$ …..(5)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a+(n-1)d$ ….. (6)

Substituting the values from (1), (3) in (6) we get,

$4 = a+2(n-1)$ …..(7)

Let us solve equations (5) and (7) by substituting the value of a from (7) in (5) we get,

$-14 = n\left[\left(4-2(n-1)\right)+n-1\right]$

$\Rightarrow -14 = n\left[5-n\right]$

$\Rightarrow n^2-5n-14=0$

$\Rightarrow n^2-7n+2n-14=0$

$\Rightarrow (n-7)(n+2)=0$

$\therefore n=7$ (Since n cannot be negative) ……(8)

Substituting the values from (8) in (7) we get,

$4 = a+2(7-1)$

$\Rightarrow 4 = a+12$

$\therefore a=-8$

ix) Given $a=3$, $n=8$, $S=192$, find d.

Ans: Given, first term, $a=3$ …..(1)

Given, the sum of terms, $S_n = 192$ ….. (2)

Given, the number of terms, $n=8$ …..(3)

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ …..(4)

Substituting the values from (1), (2) in (4) we get, $192 = \frac{8}{2}\left[2(3) + d(8-1)\right]$

$\Rightarrow 192 = 4[6 + 7d]$
$\Rightarrow 48 = 6 + 7d$
$\Rightarrow 42 = 7d$
$\therefore d = 6$

x) Given $l = 28$, $S = 144$ and there are total 9 terms. Find a.
Ans: Given, last term, $l = 28$ …..(1)
Given, the sum of terms, $S_n = 144$ ….. (2)
Given, the number of terms, $n = 9$ …..(3)
We know that the sum of n terms of the A.P. with first term a and last term l is given by $S_n = \frac{n}{2}[a + l]$ …..(4)
Substituting the values from (1), (2) in (4) we get, $144 = \frac{9}{2}[a + 28]$

$\Rightarrow 32 = a + 28$
$\therefore a = 4$

4. How many terms of the A.P. $9, 17, 25...$ must be taken to give a sum of 636 ?
Ans: Given, common difference, $d = 17 - 9 = 8$ …..(1)
Given, first term, $a = 9$ …..(2)
Given, the sum of terms, $S_n = 636$ ….. (3)
We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ …..(3)

Substituting the values from (1), (2), (3) in (4) we get, $636 = \frac{n}{2}\left[2(9) + 8(n-1)\right]$

$\Rightarrow 636 = n(5 + 4n)$
$\Rightarrow 4n^2 + 5n - 636 = 0$
$\Rightarrow 4n^2 + 53n - 48n - 636 = 0$
$\Rightarrow n(4n + 53) - 12(4n + 53) = 0$
$\Rightarrow (n - 12)(4n + 53) = 0$
$\Rightarrow n = 12 \text{ or } -\frac{53}{4}$
Since n can only be a natural number $\therefore n = 12$

5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Ans: Given, first term, $a = 5$ …..(1)

Given, the sum of terms, $S_n = 400$ ….. (2)

Given, the n^{th} term, $a_n = 45$ …..(3)

We know that the sum of n terms of the A.P. with first term a and last term l is given by $S_n = \frac{n}{2}[a + l]$ …..(4)

Substituting the values from (1), (2), (3) in (4) we get, $400 = \frac{n}{2}[5 + 45]$

$\Rightarrow 400 = 25n$

$\therefore n = 16$

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n - 1)d$ ….. (5)

Substituting the values from (1), (3) in (5) we get,

$45 = 5 + (16 - 1)d$

$\Rightarrow 40 = 15d$

$\therefore d = \frac{8}{3}$

6. The first and the last term of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Ans: Given, first term, $a = 17$ …..(1)

Given, the common difference, $d = 9$ ….. (2)

Given, the n^{th} term, $a_n = 350$ …..(3)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n - 1)d$ ….. (4)

Substituting the values from (1), (2), (3) in (4) we get,

$350 = 17 + 9(n - 1)$

$\Rightarrow 333 = 9(n - 1)$

$\Rightarrow 37 = (n - 1)$

$\therefore n = 38$ ……(5)

We know that the sum of n terms of the A.P. with first term a and last term l is given by $S_n = \frac{n}{2}[a+l]$ …..(6)

Substituting the values from (1), (5), (3) in (6) we get, $S_{38} = \frac{38}{2}[17+350]$

$\Rightarrow S_{38} = 19(367)$

$\therefore S_{38} = 6973$

7. Find the sum of first 22 terms of an AP in which $d=7$ and 22^{nd} term is 149.

Ans: Given, the common difference, $d=7$ ….. (1)

Given, the 22^{nd} term, $a_{22} = 149$ …..(2)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a+(n-1)d$ ….. (3)

Substituting the values from (1), (2) in (3) we get,

$149 = a + 7(22-1)$

$\Rightarrow 149 = a + 147$

$\therefore a = 2$ ……(4)

We know that the sum of n terms of the A.P. with first term a and last term l is given by $S_n = \frac{n}{2}[a+l]$ …..(5)

Substituting the values from (1), (2), (4) in (5) we get, $S_{22} = \frac{22}{2}[2+149]$

$\Rightarrow S_{22} = 11(151)$

$\therefore S_{22} = 1661$

8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Ans: Given, the 2^{nd} term, $a_2 = 14$ ….. (1)

Given, the 3^{rd} term, $a_3 = 18$ …..(2)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a+(n-1)d$ ….. (3)

Substituting the values from (1) in (3) we get,

$14 = a + d \quad .....(4)$

Substituting the values from (2) in (3) we get,

$18 = a + 2d \quad .....(5)$

Solving equations (4) and (5) by subtracting (4) from (5) we get,

$18 - 14 = (a + 2d) - (a + d)$

$\therefore d = 4 \quad ......(6)$

Substituting the value from (6) in (4) we get $a = 10$ ....(7)

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right] \quad .....(8)$

Substituting the values from (7), (6) in (8) we get for $n = 51$,

$S_{51} = \frac{51}{2}\left[2(10) + 4(51-1)\right]$

$\Rightarrow S_{51} = \frac{51}{2}\left[20 + 200\right]$

$\therefore S_{51} = 5610$

9. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289 , find the sum of first n terms.

Ans: Given, the sum of first 7 terms, $S_7 = 49 \quad .....(1)$

Given, the sum of first 17 terms, $S_{17} = 289 \quad .....(2)$

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right] \quad .....(3)$

Substituting the values from (1) in (3) we get,

$49 = \frac{7}{2}\left[2a + (7-1)d\right]$

$\Rightarrow 7 = a + 3d \quad .....(4)$

Substituting the values from (2) in (3) we get,

$289 = \frac{17}{2}\left[2a + (17-1)d\right]$

$\Rightarrow 17 = a + 8d \quad .....(5)$

Solving equations (4) and (5) by subtracting (4) from (5) we get,

$17 - 7 = (a + 8d) - (a + 3d)$

$\Rightarrow 10 = 5d$

$\therefore d = 2 \quad ......(6)$

Substituting the value from (6) in (4) we get $a = 1 \quad .....(7)$

Substituting the values from (7), (6) in (3) we get,

$$S_n = \frac{n}{2}\left[2 + 2(n-1)\right]$$

$\therefore S_n = n^2$

10. Show that $a_1, a_2 ..., a_n, ...$ form an AP where a_n is defined as below. Also find the sum of the first 15 terms in each case.

i) $a_n = 3 + 4n$

Ans: Consider two consecutive terms of the given sequence. Say a_n, a_{n+1}.

Difference between these terms will be

$a_{n+1} - a_n = \left[3 + 4(n+1)\right] - \left[3 + 4n\right]$

$\Rightarrow a_{n+1} - a_n = 4(n+1) - 4n$

$\Rightarrow a_{n+1} - a_n = 4$

Which is a constant $\forall n \in \mathbb{N}$.

For $n = 1$, $a_1 = 3 + 4 = 7$

Therefore, it is an A.P. with first term 7 and common difference 4.

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

Therefore, $S_{15} = \frac{15}{2}\left[2(7) + 4(15-1)\right]$

$\Rightarrow S_{15} = \frac{15}{2}\left[14(5)\right]$

$\therefore S_{15} = 525$

ii) $a_n = 9 - 5n$

Ans: Consider two consecutive terms of the given sequence. Say a_n, a_{n+1}. Difference between these terms will be

$$a_{n+1} - a_n = [9 - 5(n+1)] - [9 - 5n]$$

$$\Rightarrow a_{n+1} - a_n = -5(n+1) + 5n$$

$$\Rightarrow a_{n+1} - a_n = -5$$

Which is a constant $\forall n \in \mathbb{N}$.

For $n = 1$, $a_1 = 9 - 5 = 4$

Therefore, it is an A.P. with first term 4 and common difference -5.

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}[2a + (n-1)d]$

Therefore, $S_{15} = \frac{15}{2}\left[2(4) - 5(15-1)\right]$

$$\Rightarrow S_{15} = 15[-31]$$

$$\therefore S_{15} = -465$$

11. If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly find the 3^{rd}, the 10^{th} and the n^{th} terms.

Ans: Given, the sum of the first n terms of an A.P. is $4n - n^2$.

First term $= S_1 = 4 - 1 = 3$ ....(1)

Sum of first two terms $= S_2 = 8 - (2)^2 = 4$ …..(2)

From (1) and (2), 2^{nd} term $= S_2 - S_1 = 4 - 3 = 1$.

Sum of first three terms $= S_3 = 12 - (3)^2 = 3$ …..(3)

From (3) and (2), 3^{rd} term $= S_3 - S_2 = 3 - 4 = -1$.

Similarly,

Sum of first n terms $= S_n = 4n - n^2$ …..(4)

Sum of first $n-1$ terms $= S_{n-1} = 4(n-1) - (n-1)^2 = -n^2 + 6n - 5$ …..(5)

From (4) and (5), n^{th} term $= S_n - S_{n-1} = (4n - n^2) - (-n^2 + 6n - 5) = 5 - 2n$ …..(6)

From (6), 10^{th} term is $5 - 2(10) = -15$.

12. Find the sum of first 40 positive integers divisible by 6.

Ans: First positive integer that is divisible by 6 is 6 itself.

Second positive integer that is divisible by 6 is 6 + 6 =12.

Third positive integer that is divisible by 6 is $12 + 6 = 18$.

Hence, it is an A.P. with first term and common difference both as 6.

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

Therefore, for $n = 40$,

$$S_{40} = \frac{40}{2}\left[2(6) + 6(40-1)\right]$$

$\Rightarrow S_{40} = 120[41]$

$\therefore S_{40} = 4920$

13. Find the sum of first 15 multiples of 8.

Ans: First positive integer that is divisible by 8 is 8 itself.

Second positive integer that is divisible by 8 is 8 + 8 =16 .

Third positive integer that is divisible by 8 is $16 + 8 = 24$.

Hence, it is an A.P. with first term and common difference both as 8.

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

Therefore, for $n = 15$,

$$S_{15} = \frac{15}{2}\left[2(8) + 8(15-1)\right]$$

$\Rightarrow S_{15} = 60[16]$

$\therefore S_{15} = 960$

14. Find the sum of the odd numbers between 0 and 50.

Ans: The odd numbers between 0 and 50 are $1,3,5,...,49$.

It is an A.P. with first term 1 and common difference 2. ….(1)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$ … (2)

Substitute $a_n = 49$ and values from (1) into (2)

$49 = 1 + 2(n - 1)$

$\Rightarrow 24 = (n - 1)$

$\therefore n = 25$ ……(3)

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ …..(4)

Substituting values from (1), (3) in (4) we get,

$S_{25} = \frac{25}{2}\left[2 + 2(25 - 1)\right]$

$\Rightarrow S_{25} = 25[25]$

$\therefore S_{25} = 625$

15. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs. 250 for the second day, Rs. 300 for the third day, etc., the penalty for each succeeding day being Rs. 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

Ans: Penalty of delay for first day is Rs. 200.

Penalty of delay for second day is Rs. 250.

Penalty of delay for third day is Rs. 300.

Hence it is an A.P. with first term 200 and common difference 50.

Money the contractor has to pay as penalty, if he has delayed the work by 30 days is the sum of first 30 terms of the A.P.

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$. Therefore,

$S_{30} = \frac{30}{2}\left[2(200) + 50(30 - 1)\right]$

$\Rightarrow S_{30} = 15\left[400 + 50(29)\right]$

$\therefore S_{30} = 27750$

Therefore, the contractor has to pay Rs 27750 as penalty.

16. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

Ans: Let the first prize be of Rs. a then the second prize will be of Rs. $a - 20$, the third prize will be of Rs. $a-40$.

Therefore, it is an A.P. with first term a and common difference -20.

Given, $S_7 = 700$

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$. Therefore,

$$S_7 = \frac{7}{2}\left[2a-20(7-1)\right]$$

$$\Rightarrow 700 = 7\left[a-60\right]$$

$$\Rightarrow 100 = a-60$$

$$\therefore a = 160$$

Therefore, the value of each of the prizes was

Rs 160, Rs 140, Rs 120,Rs 100, Rs 80, Rs 60, and Rs 40.

17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students?

Ans: Each section of class I will plant 1 tree each. Therefore, total trees planted by class I are 3.

Each section of class II will plant 2 trees each. Therefore, total trees planted by class II are $3\times 2=6$.

Each section of class III will plant 3 trees each. Therefore, total trees planted by class III are $3\times 3=9$.

Therefore, it is an A.P series with first term and common difference both as 3.

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$. Therefore,

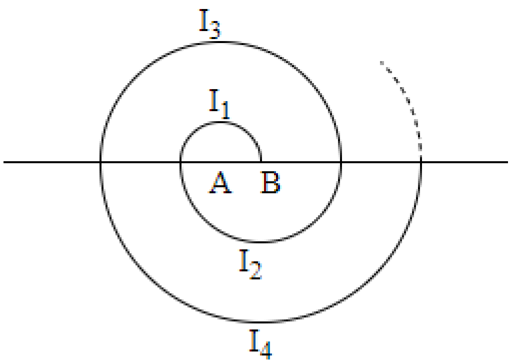
$$S_{12} = \frac{12}{2}\left[2(3)-3(12-1)\right]$$

$$\Rightarrow S_{12} = 6\left[39\right]$$

$\therefore S_{12} = 234$

Therefore, 234 trees will be planted by the students.

18. A spiral is made up of successive semicircles, with centers alternately at A and B, starting with center at A of radii 0.5, 1.0 cm, 1.5 cm, 2.0 cm, ... as shown in figure. What is the total length of such a spiral made up of thirteen consecutive semicircles?



Ans: Length of first semi-circle $I_1 = \pi(0.5)$ cm.

Length of second semi-circle $I_2 = \pi(1)$ cm.

Length of third semi-circle $I_3 = \pi(1.5)$ cm.

Therefore, it is an A.P series with first term and common difference both as $\pi(0.5)$.

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$. Therefore,

$$S_{13} = \frac{13}{2}\left[2(0.5\pi) + (0.5\pi)(13-1)\right]$$

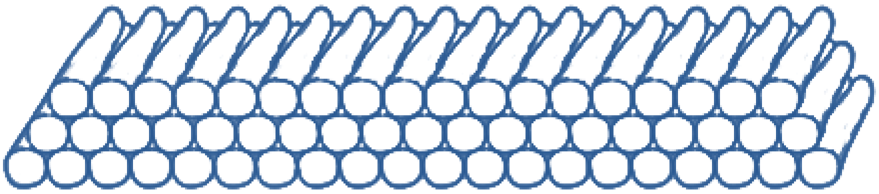
$$\Rightarrow S_{13} = 7 \times 13 \times (0.5\pi)$$

$$\Rightarrow S_{13} = 7 \times 13 \times \frac{1}{2} \times \frac{22}{7}$$

$$\therefore S_{13} = 143$$

Therefore, the length of such spiral of thirteen consecutive semi-circles will be 143 cm.

19. The 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?



Ans: Total logs in first row are 20.

Total logs in second row are 19.

Total logs in third row are 18.

Therefore, it is an A.P series with first term 20 and common difference -1.

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$. Therefore,

$$200 = \frac{n}{2}\left[2(20) - (n-1)\right]$$

$$\Rightarrow 400 = n[41 - n]$$

$$\Rightarrow n^2 - 41n + 400 = 0$$

$$\Rightarrow n^2 - 16n - 25n + 400 = 0$$

$$\Rightarrow n(n-16) - 25(n-16) = 0$$

$$\Rightarrow (n-16)(n-25) = 0$$

For $n = 25$, after 20^{th} term, all terms are negative, which is illogical as terms are representing the number of logs and number of logs being negative is illogical.

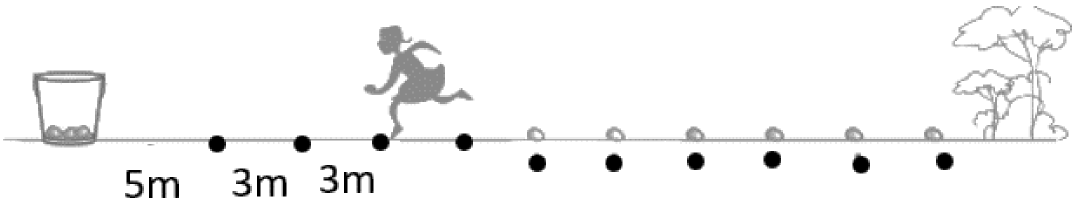
$\therefore n = 16$

Total logs in 16^{th} row $= 20 - (16-1) = 5$

Therefore, 200 logs will be placed in 16 rows and the total logs in 16^{th} row will be 5.

20. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato and other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line.
A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[Hint: to pick up the first potato and the second potato, the total distance (in metres) run by a competitor is $2\times 5+2\times(5+3)$]



Ans: Total distance run by competitor to collect and drop first potato $=2\times 5=10$ m.
Total distance run by competitor to collect and drop second potato $=2\times(5+3)=16$ m.
Total distance run by competitor to collect and drop third potato $=2\times(5+3+3)=22$ m.
Therefore, it is an A.P series with first term 10 and common difference 6.
We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$. Therefore, to collect and drop 10 potatoes total distance covered is

$S_{10}=\frac{10}{2}\left[2(10)+6(10-1)\right]$

$\Rightarrow S_{10}=5[74]$

$\therefore S_{13}=370$

Therefore, the competitor will run a total distance of 370 m.

Exercise 5.4

1. Which term of the A.P. 121,117,113,... is its first negative term?
[Hint: Find n for $a_n<0$]

Ans: Given A.P. $121, 117, 113, ...$

Its first term is 121 and common difference is $117 - 121 = -4$.

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$.

Therefore the n^{th} term of the given A.P. is $a_n = 121 - 4(n-1)$ ….. (1)

To find negative term, find n such that $a_n < 0$

Hence from (1),

$121 - 4(n-1) < 0$

$\Rightarrow 121 < 4(n-1)$

$\Rightarrow \frac{121}{4} + 1 < n$

$\Rightarrow n > \frac{125}{4}$

$\therefore n > 31.25$

Therefore, the 32^{nd} term of the given A.P. will be its first negative term.

2. The sum of the third and the seventh terms of an A.P is 6 and their product is 8. Find the sum of first sixteen terms of the A.P.

Ans: Given the sum of third and seventh term of A.P., $a_3 + a_7 = 6$ …..(1)

Given the sum of third and seventh term of A.P., $a_3 \cdot a_7 = 8$ …..(2)

We know that the n^{th} term of the A.P. with first term a and common difference d is given by $a_n = a + (n-1)d$. Therefore,

For $n = 3, a_3 = a + 2d$

For $n = 7, a_7 = a + 6d$

From (1), $a_3 + a_7 = (a + 2d) + (a + 6d)$

$\Rightarrow 2a + 8d = 6$

$\therefore a + 4d = 3$ ….. (3)

From (2), $a_3 \cdot a_7 = (a + 2d) \cdot (a + 6d)$

$\therefore a^2 + 8ad + 12d^2 = 8$ …..(4)

Let us now solve equations (3) and (4) by substituting the value of a from (3) into (4).

$(3 - 4d)^2 + 8d(3 - 4d) + 12d^2 = 8$

$\Rightarrow 9 - 24d + 16d^2 + 24d - 32d^2 + 12d^2 = 8$

$\Rightarrow -4d^2 + 1 = 0$

$\Rightarrow d^2 = \frac{1}{4}$

$\therefore d = \frac{1}{2}, -\frac{1}{2}$ …..(5)

<u>CASE 1:</u> For $d = \frac{1}{2}$

Substitute $d = \frac{1}{2}$ in (6) we get, $a = 1$ …..(6)

Therefore, it is an A.P series with first term 1 and common difference $\frac{1}{2}$.

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$. Therefore,

$$S_{16} = \frac{16}{2}\left[2 + \frac{1}{2}(16-1)\right]$$

$\Rightarrow S_{16} = 4[19]$

$\therefore S_{16} = 76$

<u>CASE 2:</u> For $d = -\frac{1}{2}$

Substitute $d = -\frac{1}{2}$ in (6) we get, $a = 5$ …..(7)

Therefore, it is an A.P series with first term 5 and common difference $-\frac{1}{2}$ and hence,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

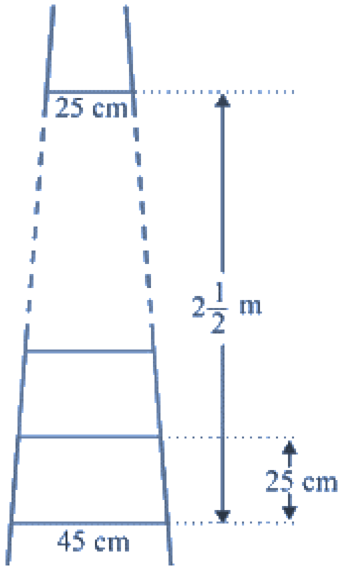
$$\Rightarrow S_{16} = \frac{16}{2}\left[2(5) - \frac{1}{2}(16-1)\right]$$

$\Rightarrow S_{16} = 4[5]$

$\therefore S_{16} = 20$

3. A ladder has rungs 25 cm apart. (See figure). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?

[Hint: number of rungs $=\frac{250}{25}$]



Ans: Distance between first and last rungs is $2\frac{1}{2}\text{m}=\frac{5}{2}\text{m}=250\text{cm}$.

Distance between two consecutive rungs is 25cm.

Therefore, total number of rungs are $\frac{250}{25}+1=11$.

Also, we can observe that the length of each rung is decreasing in a uniform order. So, we can conclude that the length of rungs is in A.P. with first term 45, common difference -25 and number of terms 11.

We know that the sum of n terms of the A.P. with first term a and last term l is given by $S_n=\frac{n}{2}\left[a+l\right]$. Therefore,

$S_{11}=\frac{11}{2}\left[45+25\right]$

$\Rightarrow S_{11}=11\left[35\right]$

$\therefore S_{11}=385$

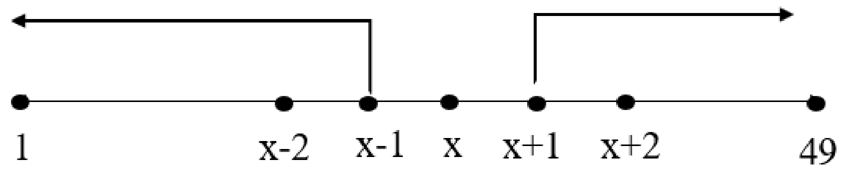
Therefore, the length of the wood required for the rungs is 385cm.

4. The houses of a row are number consecutively from 1 to 49. Show that there is a value of x such that the sum of numbers of the houses preceding the house numbered x is equal to the sum of the number of houses following it. Find this value of x.
[Hint $S_{x-1} = S_{49} - S_x$]

Ans: Given houses are numbered 1,2,3,4,....

Clearly, they are numbered in A.P. series with both first term and common difference as 1.

Now, there is house numbered x such that the sum of numbers of the houses preceding the house numbered x is equal to the sum of the number of houses following it i.e., $S_{x-1} = S_{49} - S_x$



We know that the sum of n terms of the A.P. with first term a and last term l is given by $S_n = \frac{n}{2}[a + l]$. Therefore,

$$S_{x-1} = S_{49} - S_x$$

$$\Rightarrow \left\{\frac{(x-1)}{2}\left[1+(x-1)\right]\right\} = \left\{\frac{49}{2}[1+49]\right\} - \left\{\frac{x}{2}[1+x]\right\}$$

$$\Rightarrow \frac{x(x-1)}{2} = 49[25] - \frac{x(x+1)}{2}$$

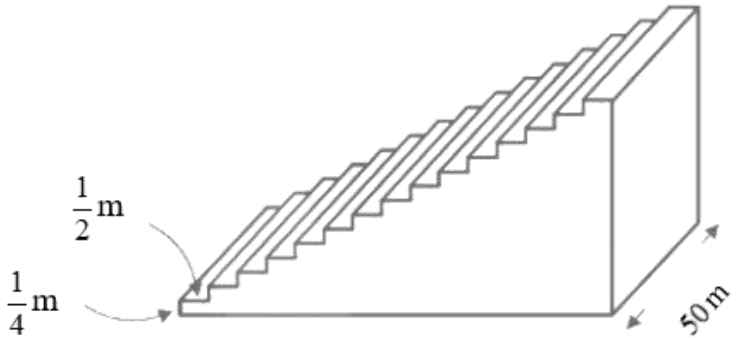
$$\Rightarrow x(x-1) = 2450 - x(x+1)$$

$$\Rightarrow 2x^2 = 2450$$

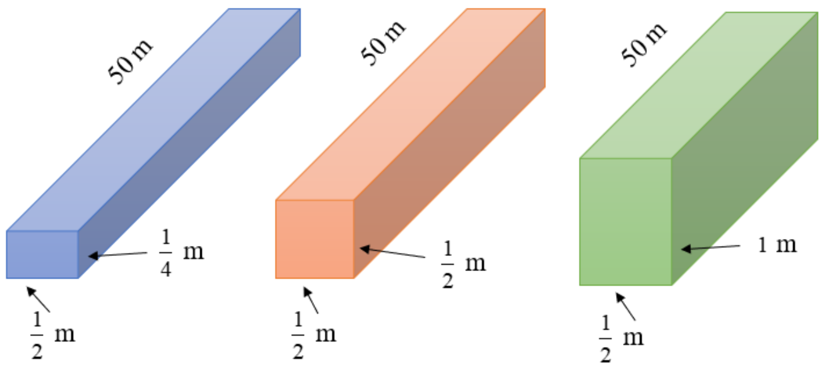
$\therefore x = 35$ (Since house number cannot be negative)

Therefore, house number 35 is such that the sum of the numbers of houses preceding the house numbered 35 is equal to the sum of the numbers of the houses following it.

5. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has a rise of $\frac{1}{4}$m and a tread of $\frac{1}{2}$m (See figure) calculate the total volume of concrete required to build the terrace.

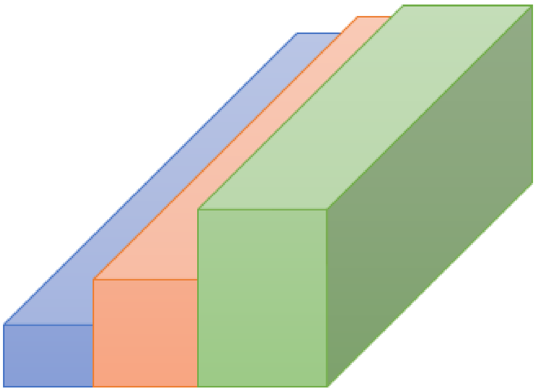


Ans: Given that a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has a rise of $\frac{1}{4}$m and a tread of $\frac{1}{2}$m**.**
An easy illustration of the problem is depicted below.



Here blue step is the lowermost step. Let it be known as step 1. The volume of step 1 is $\frac{1}{2}\times\frac{1}{4}\times 50\text{ m}^3$.
The red step is the second lowermost step. Let it be known as step 2. The volume of step 2 is $\frac{1}{2}\times\frac{1}{2}\times 50\text{ m}^3$.

The green step is the third lower step. Let it be known as step 3. The volume of step 3 is $\frac{1}{2}\times 1\times 50\ m^3$.



We can see that the height is increasing with each increasing step by a factor of $\frac{1}{4}$, length and width being constant. Hence the volume of each step is increasing by $\frac{1}{2}\times\frac{1}{4}\times 50\ m^3$.

Therefore, we can conclude that the volume of steps is in A.P. with first term and common difference both as $\frac{1}{2}\times\frac{1}{4}\times 50=\frac{25}{4}\ m^3$.

We know that the sum of n terms of the A.P. with first term a and common difference d is given by $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$. Therefore,

$$S_{15}=\frac{15}{2}\left[2\left(\frac{25}{4}\right)+\left(\frac{25}{4}\right)(15-1)\right]$$

$$\Rightarrow S_{15}=\frac{15}{2}\left(\frac{25}{4}\right)[16]$$

$$\Rightarrow S_{15}=15\cdot 25\cdot 2$$

$$\therefore S_{15}=750$$

Therefore, volume of concrete required to build the terrace is $750\ m^3$.