Maths Class 10 NCERT Solutions

Chapter 13 – Surface areas And Volumes

<u>Exercise 13.1</u>

1. 2 cubes of each of volume $64\ \text{cm}^3$ are joined end to end. Find the surface area of the resulting cuboids.

Ans: Given that,

2 cubes are joined end to end as given in the following diagram.

To find the surface area of the resulting cuboid.

Volume of each cube $= 64\ \text{cm}^3$

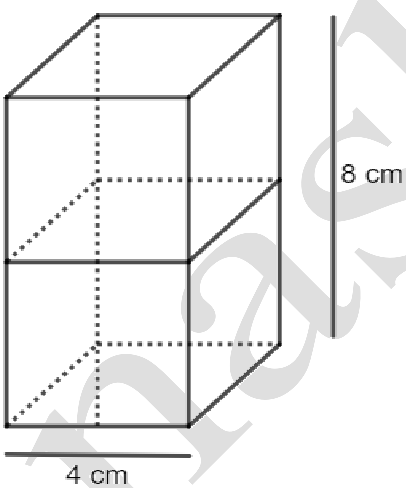
We know that,

Volume of a cube $= \text{a}^3$

$\text{a}^3 = 64$

$\text{a} = 4\ \text{cm}$

Thus the dimension of the resulting cuboid is of $4\ \text{cm}$, $4\ \text{cm}$ and $8\ \text{cm}$ when they are joined end to end. That is, $\text{l} = 4\ \text{cm}$, b=4 cmand $\text{h} = 8\ \text{cm}$



Then,

Surface area of cuboid $= 2\ \ \text{lb} + \text{bh} + \text{lh}$

$= 2\ \ \ 4\times4\ +\ 4\times8\ +\ 4\times8$

$= 2\ 16 + 32 + 32$

$= 2\ 80$

$= 160\ \text{cm}^2$

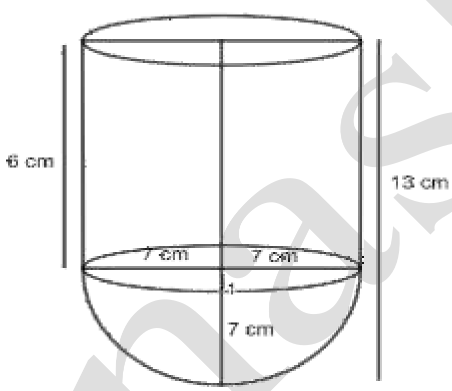
$\therefore$ The surface area of the resultant cuboid is $160\ \text{cm}^2$

2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is $14\ \text{cm}$ and the total height of the vessel is $13\ \text{cm}$. Find the inner surface area of the vessel. [Use $\pi = \frac{22}{7}$]

Ans: Given that,
The diameter of the hemisphere $= 14\ \text{cm}$
The total height of the vessel $= 13\ \text{cm}$
To find,
The inner surface area of the vessel.

Thus the radius of the hollow hemisphere $= \frac{d}{2}$

$= \frac{14}{2}$
$= 7\ \text{cm}$



From the diagram, it can be observed that the radius of the cylindrical part and that of the hemispherical part is the same.
Thus, height of hemispherical part $=$ Radius $= 7\ \text{cm}$
Height of the cylindrical part $= 13 - 7$
$= 6\ \text{cm}$
Inner surface area of the vessel $=$ CSA of cylindrical part + CSA of hemispherical part

CSA of cylindrical part $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 7 \times 6$

$= 2 \times 22 \times 6$

$= 44 \times 6$

CSA of hemispherical part $= 2\pi r^2$

$= \left(2 \times \frac{22}{7} \times 7^2\right)$

$= 2 \times 22 \times 7$

$= 44 \times 7$

Inner surface area of the vessel $= 44\ \ 6 + 7$

$= 44\ \ 13$

$= 572\ \text{cm}^2$

$\therefore$ The inner surface area of the vessel is $572\ \text{cm}^2$

3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of the same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy. [Use $\pi = \frac{22}{7}$]

Ans: Given that,

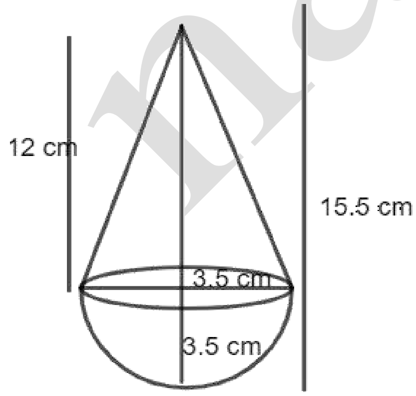
Radius of the cone $= 3.5$ cm

Radius of the hemisphere $= 3.5$ cm

The total height of the toy $= 15.5$ cm

To find,

The total surface area of the toy



From the diagram, it can be observed that the radius of both conical and hemispherical parts is the same.

Height of the hemispherical part $= 3.5$ cm

$= \frac{7}{2}$

Height of the conical part $= 15.5 - 3.5$

$= 12$ cm

Slant height of the conical part $l = \sqrt{r^2 + h^2}$

$= \sqrt{\left(\frac{7}{2}\right)^2 + 12^2}$

$= \sqrt{\left(\frac{49}{4}\right) + 144}$

$= \sqrt{\frac{625}{4}}$

$= \frac{25}{2}$

Total surface area of the toy $=$ CSA of conical part + CSA of hemispherical part

CSA of conical part $= \pi r l$

$= \left(\frac{22}{7} \times \frac{7}{2} \times \frac{25}{2}\right)$

$= 137.5$

CSA of hemispherical part $= 2\pi r^2$

$= \left(2 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2\right)$

$= 77$

Total surface area of the toy $= 137.5 + 77$

$= 214.5 \text{ cm}^2$

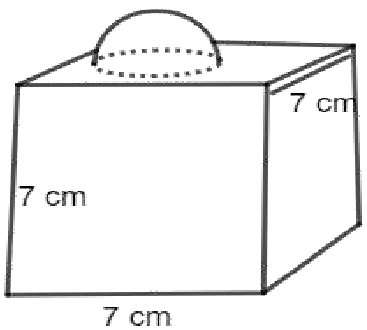
$\therefore$ The total surface area of the toy is 214.5 cm^2.

4. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

[Use $\pi = \frac{22}{7}$]

Ans: Given that,
Side of a cube $= 7$ cm
To find,

- The greatest diameter of the hemisphere.
- The surface area of the solid.



It can be observed from the diagram that the greatest possible diameter of the hemisphere is equal to the cube's edge.
Thus the greatest diameter of the hemispherical part $= 7$ cm

So the radius of the hemispherical part $= \frac{7}{2}$

$= 3.5$ cm
Total surface area of the solid = Surface area of cubical part + CSA of hemispherical part – Area of hemispherical part

$= 6a^2 + 2\pi r^2 - \pi r^2$

$= 6a^2 + \pi r^2$

$= 6(7)^2 + \left(\frac{22}{7} \times \left(\frac{7}{2}\right)^2\right)$

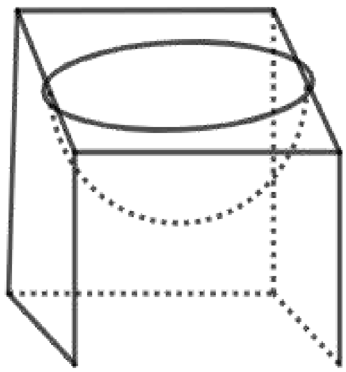
$= 294 + 38.5$

$= 332.5 \text{ cm}^2$

$\therefore$ The greatest diameter of the hemisphere is 7 cm and the surface area of the solid is 332.5 cm^2.

5. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Ans: Given that,
Diameter of the hemisphere = Edge of the cube
To determine,
The surface area of the remaining solid.



Diameter of the hemisphere = Edge of the cube $= l$

Radius of the hemisphere $= \frac{l}{2}$

Total surface area of the solid = Surface area of cubical part + CSA of hemispherical part – Area of base of hemispherical part

$= 6a^2 + 2\pi r^2 - \pi r^2$

$= 6a^2 + \pi r^2$

$= 6l^2 + \pi \left(\frac{l}{2}\right)^2$

$= 6l^2 + \frac{\pi l^2}{4}$

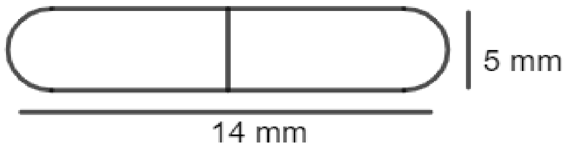
$= \frac{24l^2 + \pi l^2}{4}$

$= \frac{1}{4}(24 + \pi) l^2$ unit^2

$\therefore$ The area of the remaining solid is $\frac{1}{4}(24 + \pi) l^2$ unit^2.

6. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see the given figure). The length of the entire capsule is

14 mm and the diameter of the capsule is 5 mm. Find its surface area. [Use $\pi = \frac{22}{7}$]



Ans: Given that,
Length of the capsule $= 15$ mm
Diameter of the capsule $= 5$ mm
To find,
The surface area of the capsule
From the diagram,
Radius of cylindrical part = Radius of hemispherical part

$$= \frac{\text{Diameter of the capsule}}{2}$$

$$= \frac{5}{2}$$

Length of the cylindrical part (h) = Length of the entire capsule - 2r

$$= 14 - 2\left(\frac{5}{2}\right)$$

$$= 14 - 5$$

$$= 9 \text{ mm}$$

Surface area of capsule = 2CSA of hemispherical part + CSA of cylindrical part
2CSA of hemispherical part $= 2\ \ 2\pi r^2$

$$= 4\pi\left(\frac{5}{2}\right)^2$$

$$= 25\pi$$

CSA of cylindrical part $= 2\pi rh$

$$= 2\pi\left(\frac{5}{2}\right)\times 9$$

$$= 45\pi$$

Surface area of the capsule $= 25\pi + 45\pi$

$$= 70\pi$$

$= 70 \times \frac{22}{7}$

$= 10 \times 22$

$= 220 \text{ mm}^2$

$\therefore$ The surface area of the capsule is 220 mm^2.

7. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs.500 per m^2. (Note that the base of the tent will not be covered with canvas.)

Ans: Given that,

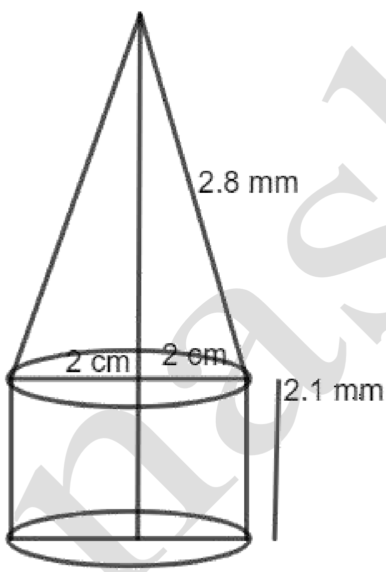
Height of the cylindrical part $= 2.1$ m

Diameter of the cylindrical part $= 4$ m

Radius of the cylindrical part $= 2$ m

Slant height of conical part $= 2.8$ m

Cost of 1 m^2 canvas $=$ Rs.500



Area of the canvas used = CSA of conical part + CSA os cylindrical part

$= \pi rl + 2\pi rh$

$= \pi \times 2 \times 2.8 + 2\pi \times 2 \times 2.1$

$= 2\pi \ 2.8 + 4.2$

$= 2 \times \frac{22}{7} \times 7$

$= 44 \text{ m}^2$

Cost of 1 m^2 canvas $=$ Rs.500

Cost of 44 m^2 canvas $= 44 \times 500$

$=$ Rs.22,000

$\therefore$ The cost the canvas that is used to cover the tent is Rs.22,000.

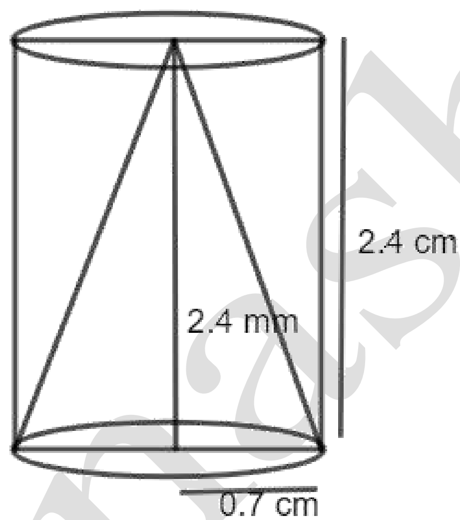
8. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2. [Use $\pi = \frac{22}{7}$]

Ans: Height of the cylindrical part = Height of the conical part $= 2.4$ cm

Diameter of the cylindrical part = Diameter of the conical part $= 1.4$ cm

To find,

The total surface area of the remaining solid.



Diameter of the cylindrical part $= 1.4$ cm

Radius of the cylindrical part $= \frac{1.4}{2}$

$= 0.7$ cm

Slant height of conical part $= \sqrt{r^2 + h^2}$

$=\sqrt{0.7^{2}+2.4^{2}}$
$=\sqrt{0.49+5.76}$
$=\sqrt{6.25}$
$=2.5$

The total surface area of the solid = CSA of cylindrical part + CSA of conical part + Area of cylindrical base

$=2\pi rh+\pi rl+\pi r^2$

$=\left(2\times\frac{22}{7}\times 0.7\times 2.4\right)+\left(\frac{22}{7}\times 0.7\times 2.5\right)+\left(\frac{22}{7}\times 0.7^{2}\right)$

$=4.4\times 2.4+2.2\times 2.5+2.2\times 0.7$

$=10.56+5.5+1.54$

$=17.6\ cm^2$

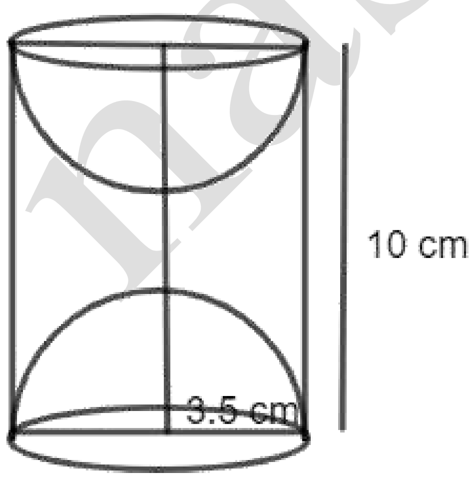
$\therefore$ The total surface area of the remaining solid to the nearest cm^2 is 18 cm^2.

9. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in the given figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm, find the total surface area of the article. [Use $\pi=\frac{22}{7}$]

Ans: Given,

Height of the cylindrical part $=10$ cm

Radius of the cylindrical part = Radius of the hemispherical part $=3.5$



The total surface area of the article = CSA of cylindrical part + 2CSA of hemispherical part

$= 2\pi rh + 2 \times 2\pi r^2$

$= 2\pi \times 3.5 \times 10 + 4\pi \ 3.5^2$

$= 70\pi + 49\pi$

$= 119\pi$

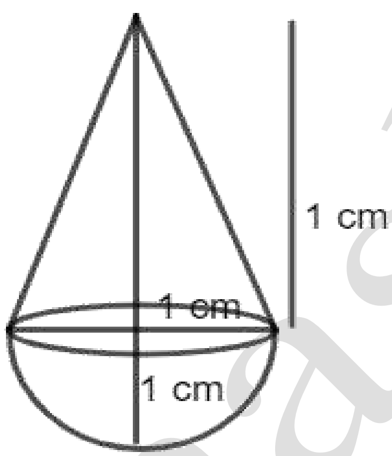
$= 119\left(\frac{22}{7}\right)$

$= 374 \text{ cm}^2$

$\therefore$ The total surface area of the article is 374 cm^2.

Exercise 13.2

1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.



Ans: Given that,

Radius of cone and the hemisphere $= 1$ cm

Height of the cone = Radius of the cone $= 1$ cm

To find,

The volume of the solid in terms of π.

Volume of the given solid = Volume of the conical solid + Volume of the Hemispherical solid

$\frac{1}{3}\ r^2h\ \ \frac{2}{3}\ r^3$

$\equiv \frac{1}{3}\pi\ 1^2 + 1 - + \pi \frac{2}{3}\pi\ 1^3$

$= \frac{1}{3}\pi + \frac{2}{3}\pi$

$= \frac{3}{3}\pi$

$= \pi$

$\therefore$ The volume of the solid in terms of π is $\pi\ cm^3$.

2. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminum sheet. The diameter of the model is 3 cm and its length is 12 cm . If each cone has a height of 2 cm , find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same). [Use $\pi = \frac{22}{7}$]

Ans: Given that,

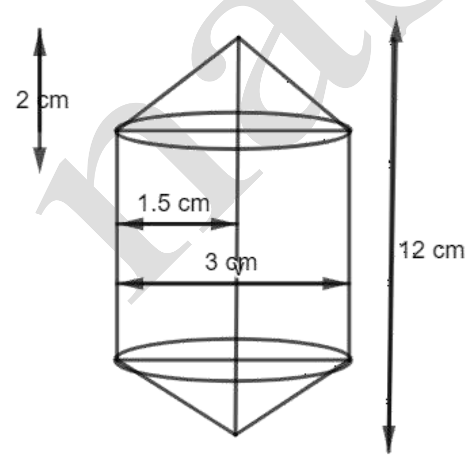
Diameter of the cylindrical part $= 3$ cm

Length of the cylindrical part $= 12$ cm

Height of the conical part $= 2$ cm

To find,

The volume of the air contained in the model



It can be observed from the figure that the,

Height of each conical part $h_1 = 2$ cm

Height of cylindrical part $h_2 = 12 - 2 \times$ Height of the conical part

$= 12 - 2 \times 2$

$= 12 - 4$

$= 8$ cm

Diameter of the cylindrical part $= 3$ cm

Radius of the cylindrical part = Radius of the conical part

$= \frac{3}{2}$

Volume of the air in the model = Volume of cylindrical part + 2(Volume of cones)

Volume of the cylinder $= \pi r^2 h_2$

$= \pi \left(\frac{3}{2}\right)^2 8$

$= \pi \times \frac{9}{4} \times 8$

$= 18\pi$

Volume of 2 cones $= 2 \times \frac{1}{3} \pi r^2 h$

$= 2 \times \frac{1}{3} \pi \times \left(\frac{3}{2}\right)^2 \times 2$

$= \frac{2}{3} \pi \times \frac{9}{4} 2$

$= 3\pi$

Volume of the air in cuboid $= 18\pi + 3\pi$

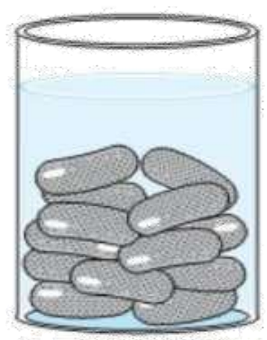
$= 21\pi$

$= 21\left(\frac{22}{7}\right)$

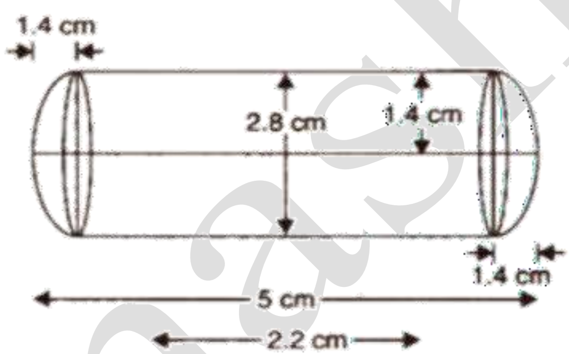
$= 66 \text{ cm}^3$

3. Ag gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each

shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm(see the given figure). [Use $\pi = \frac{22}{7}$]



Ans: Given that,
The length of the gulab jamun $= 5$ cm
Diameter of the gulab jamun $= 2.8$ cm
The volume of syrup in gulab jamun is 30% to its volume.
To find,
The volume of syrup in 45 gulab jamun.
The diagram of gulab jamun shaped like a cylinder with two hemispherical ends is shown in the following diagram.



From the diagram,
Radius of the cylindrical part r_1 = Radius of hemispherical part r_2

$$= \frac{2.8}{2}$$

$$= 1.4 \text{ cm}$$

The length of the hemispherical part is the same as that of the radius of the hemispherical part.

Length of each hemispherical part $=1.4$ cm

Height of the cylindrical part $=5-2\times$Length of hemispherical part

$=5-2\ 1.4$

$=5-2.8$

$=2.2$ cm

Volume of one gulab jamun = Volume of cylindrical part $+2$(Volume of hemispherical part

Volume of cylindrical part $=\pi r^2 h$

$=\pi\ 1.4^{\ 2}\ \ 2.2$

$=\dfrac{22}{7}\ 1.4^{\ 2}\times 2.2$

$=13.552$

2 (Volume of hemispherical part) $=2\times\dfrac{2}{3}\pi r^3$

$=\dfrac{4}{3}\pi r^3$

$=\dfrac{4}{3}\pi\ 1.4^{\ 3}$

$=\dfrac{4}{3}\times\dfrac{22}{7}\times\ 1.4^{\ 3}$

$=11.498$

Volume of one gulab jamun $=13.552+11.498$

$=25.05\ \text{cm}^3$

Volume of 45 gulab jamuns $=45\times 25.05$

$=1{,}127.25\ \text{cm}^3$

Volume of sugar syrup $=30\%$ of volume

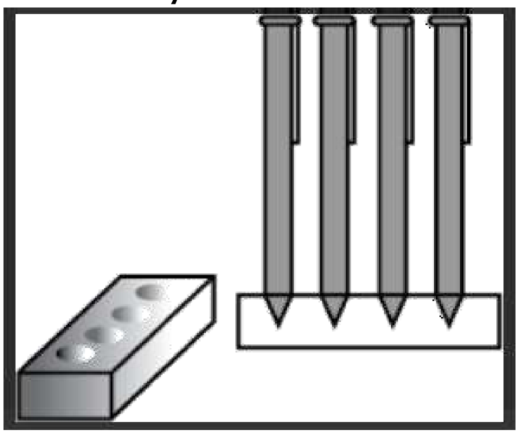
$=\dfrac{30}{100}\times 1127.25$

$=338.17\ \text{cm}^3$

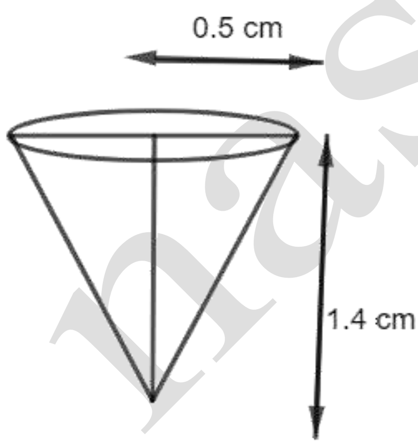
$\therefore$ The volume of sugar syrup found in 45 gulab jamuns is approximately $338\ \text{cm}^3$.

4. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboids are 15 cm by 10 cm

by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see the following figure0. [Use $\pi = \frac{22}{7}$]



Ans: Length of the cuboid $= 15$ cm
Breadth of the cuboid $= 10$ cm
Height of the cuboid $= 3.5$ cm
Radius of conical depression $= 0.5$ cm
Height of conical depression $= 1.4$ cm
To find,
The volume of wood in entire stand



Volume of the wood = Volume of cuboid - $4 \times$ Volume of cones
Volume of cuboid $= \text{lbh}$
$= 15 \times 10 \times 3.5$
$= 525 \text{ cm}^3$

Volume of cones $= 4 \times \frac{1}{3}\pi r^2 h$

$= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{1}{2}\right)^2 \times 1.4$

$= 1.47\ \text{cm}^3$

Volume of the wood $= 525 - 1.47$

$= 523.53\ \text{cm}^3$

$\therefore$ The volume of the wood in the entire stand is $523.53\ \text{cm}^3$.

5. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Ans: Given that,

Height of the conical vessel $h = 8$ cm

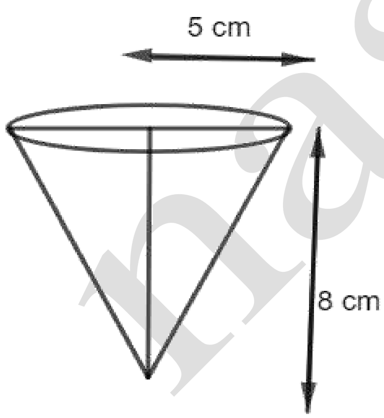
Radius of conical vessel $r_1 = 5$ cm

Radius of lead shots $r_2 = 0.5$ cm

One-fourth of water flows out from the vessel.

To find,

The number of lead shots dropped in the vessel.



Let the number of lead shots that has been dropped in the vessel be n.

Volume of water flows out = Volume of lead shots dropped in the vessel

$\frac{1}{4} \times \text{Volume of the cone} = n \times \frac{4}{3}\pi r_2^3$

$$\frac{1}{4}\times\frac{1}{3}\pi r_1^2 h = n\times\frac{4}{3}\pi r_2^3$$

$$r_1^2 h = 16 n r_2^3$$

Substituting the values we known, we obtain,

$$5^2\ 8 = n\ 16\times\ 0.5^{\ 3}$$

$$n = \frac{200}{16\times\ 0.5^{\ 3}}$$

$$n = 100$$

$\therefore$ The number of lead shots dropped in the vessel is 100.

6. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cmand radius 8 cm. Find the mass of the pole, given that $1\ cm^3$ of iron has approximately 8 g mass. [Use $\pi = 3.14$]

Ans: Let there be two cylinders, one is of larger and the other is smaller

Given that,

Height of the larger cylinder $h_1 = 220$ cm

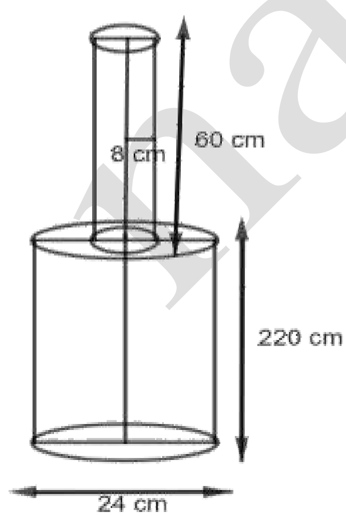
Diameter of the larger cylinder $d_1 = 24$ cm

Height of the smaller cylinder $h_2 = 60$ cm

Radius of the smaller cylinder $r_2 = 8$ cm

To find,

The mass of the iron pole.



Radius of larger cylinder $r_1 = \frac{24}{2}$

$= 12$ cm

Total volume of pole = Volume of the larger cylinder + Volume of the smaller cylinder

Volume of the larger cylinder $= \pi {r_1}^2 h_1$

$= \pi\ 12^{\ 2} \times 220$

$= 31{,}680\pi$

Volume of the smaller cylinder $= \pi {r_2}^2 h_2$

$= 3840\pi$

Volume of the iron pole $= 31{,}680\pi + 3{,}840\pi$

$= 35{,}520\pi$

$= 35{,}520\ 3.14$

$= 111{,}532.8\ \text{cm}^3$

Mass of 1 cm^3 iron $= 8$ g

Mass of $111{,}532.8\ \text{cm}^3$ iron $= 111{,}532.8 \times 8$

$= 892262.4$ g

We know that 1000 g $= 1$ kg

$892262.4\ \text{g} = 892262.4 \times 1000$ kg

$= 892.262$ kg

$\therefore$ The mass of the iron is 892.262 kg.

7. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of the water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm. [Use $\pi = \frac{22}{7}$].

Ans: Given that,

A solid with a right circular cone and a hemisphere.

Height of the conical part of the cylinder $= 120$ cm

Radius of the conical part of the cylinder $= 60$ cm

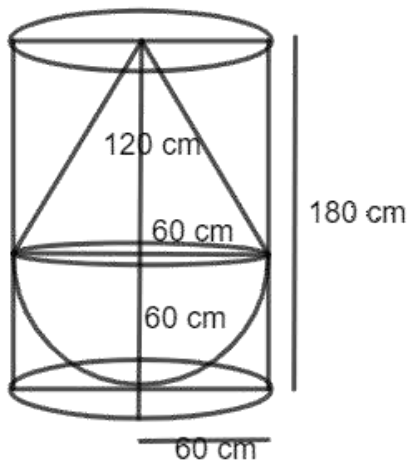
Radius of hemispherical part of the cylinder $= 60$ cm

Radius of the outer cylinder $= 60$ cm

Height of the outer cylinder $= 180$ cm

To find,

The volume of the water left in the cylinder.



Volume of the water left = Volume of cylinder – Volume of the solid

Volume of the cylinder $= \pi r^2 h$

$= \pi \; 60^{2} \; 180$

$= \pi \times 3600 \times 180$

$= 648{,}000\pi \text{ cm}^3$

Volume of the solid = Volume of cone + Volume of the hemisphere

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi \; 60^{2} 120 + \frac{2}{3}\pi \; 60^{3}$

$= \frac{1}{3}\pi \; 432{,}000 + \frac{2}{3}\pi \; 216{,}000$

$= 288{,}000\pi$

Volume of the water left $= 648{,}000\pi - 288{,}000\pi$

$= 360{,}000\pi$

$= 360{,}000 \times \frac{22}{7}$

$= 11311428.57 \text{ cm}^3$

$= 1.131 \text{ m}^3$

$\therefore$ The volume of the water left in the cylinder is 1.131 m^3

8. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child find its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurement and $\pi = 3.14$

Ans: Given that,

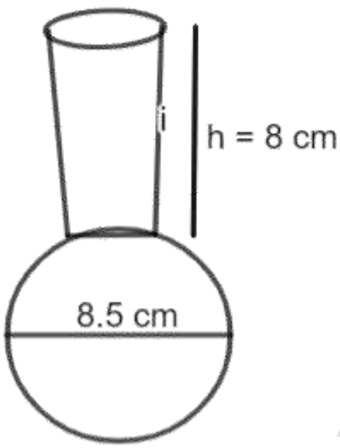
Height of the cylindrical part $= 8\text{ cm}$

Diameter of cylindrical neck $= 2\text{ cm}$

Diameter of spherical glass vessel $= 8.5\text{ cm}$

Volume of the water that the vessel holds $= 345\text{ cm}^3$

To find it the above given volume is correct.



Volume of the vessel = Volume of sphere + Volume of cylinder

Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times 3.14 \times \left(\frac{8.5}{2}\right)^3$

$= 321.392\text{ cm}^3$

Volume of cylinder $= \pi r^2 h$

$= 3.14 \times 1^2 \times 8$

$= 25.12\text{ cm}^3$

Volume of the vessel $= 321.392 + 25.12$

$= 346.51\text{ cm}^3$

$\therefore$ The volume of the vessel is 346.51 cm^3 and hence the child is wrong.

<u>Exercise 13.3</u>

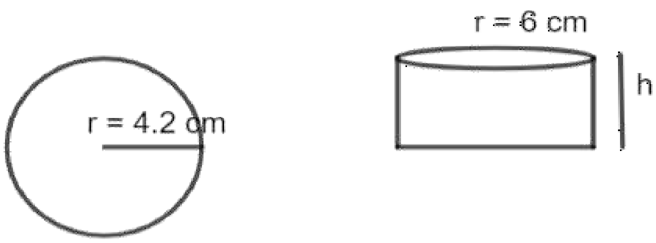
1. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Ans: Radius of the sphere $= 4.2$ cm

Radius of the cylinder $= 6$ cm

To find,

The height of the cylinder.



Let the height of the cylinder be h.

Since the spherical object has been melted and recast into a cylinder, The volumes of both the solids will be equal.

$$\frac{4}{3}\pi r^3 = \pi r^2 h$$

$$\frac{4}{3}\ 4.2^{3} = 6^{2}\ h$$

$$h = \frac{4\times 4.2\times 4.2\times 4.2}{3\times 36}$$

$$h = 2.74 \text{ cm}$$

$\therefore$ The height of the hollow cylinder is 2.74 cm.

2. Metallic spheres of radii 6 cm, 8 cm and 10 cm respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Ans: Let r_1, r_2, r_3 denote the radii of the first, second and third sphere respectively.

Given that,

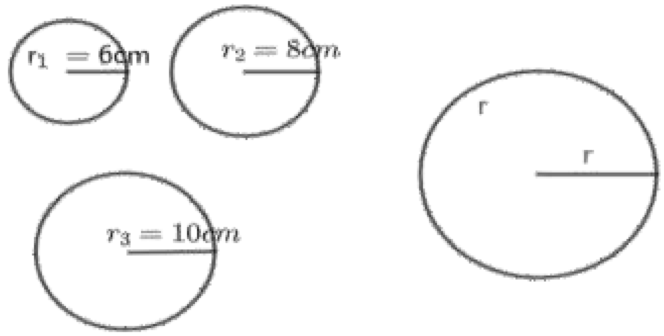
Radius of the first sphere $r_1 = 6$ cm

Radius of the second sphere $r_2 = 8$ cm

Radius of the third sphere $r_3 = 10$ cm

To find,

The radius of the resultant sphere.



Let the radius of the resultant sphere be r.
Since these spheres have been recast, the volume of the resultant solid will be the same as that of the volume of these three spheres.
Sum of the volume of three spheres = Volume of the resultant sphere

$$\frac{4}{3}\pi \ r_1^3 + r_2^3 + r_3^3 \ = \frac{4}{3}\pi r^3$$

$$6^3 + 8^3 + 10^3 \ = r^3$$

$$r^3 = 216 + 512 + 1000$$

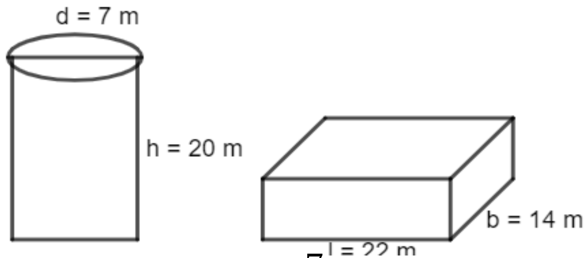
$$r^3 = 1728$$

$$r = 12 \text{ cm}$$

$\therefore$ The radius of the resultant solid sphere is 12 cm

3. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform. [Use $\pi = \frac{22}{7}$]

Ans: Given that,
Diameter of the well $= 7$ m
Height of the well $= 20$ m
It forms the platform of 22 m by 14 m.
To find,
The height of the platform.



Radius of the well $= \frac{7}{2}$ m

Let H denotes the height of the platform

Volume of the soil dug from the well will be equal to the volume of the soil on the platform.

Volume of the soil from well = Volume of the soil to make platform

$\pi r^2 h = lbH$

$\pi \left(\frac{7}{2}\right)^2 20 = 22 \times 14 \times H$

$H = \frac{22 \times 49 \times 20}{7 \times 4 \times 22 \times 14}$

$H = \frac{5}{2}$ m

$= 2.5$ m

$\therefore$ The height of the platform is 2.5 m

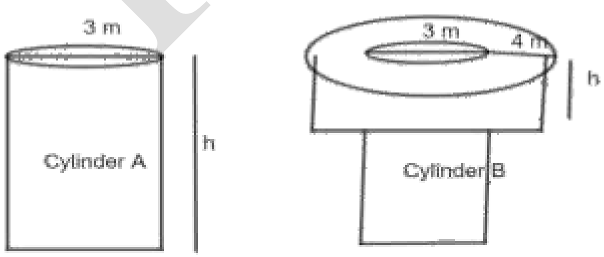
4. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Ans: Diameter of the well $= 3$ m

Height of the well $= 14$ m

Width of embankment $= 4$ m

To find the height of the embankment.



It can be observed from the figure that the embankment will in the shape of cylindrical shape with outer radius $r_2 = 4 + \frac{3}{2} = \frac{11}{2}$ and inner radius $r_1 = \frac{3}{2}$ m.

Let the height of embankment be h_2.

Volume of the soil from well = Volume of the soil in embankment

$$\pi r^2 h_1 = \pi \ r_2^2 - r_1^2 \ h_2$$

$$\left(\frac{3}{2}\right)^2 14 = \left[\left(\frac{11}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right] h_2$$

$$\frac{14 \times 9}{4} = \frac{112}{4} h_2$$

$$h_2 = \frac{9}{8}$$

$$h_2 = 1.125 \text{ m}$$

$\therefore$ The height of the embankment is 1.125 m.

5. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Ans: Height of the cylinder $h_1 = 15$ cm

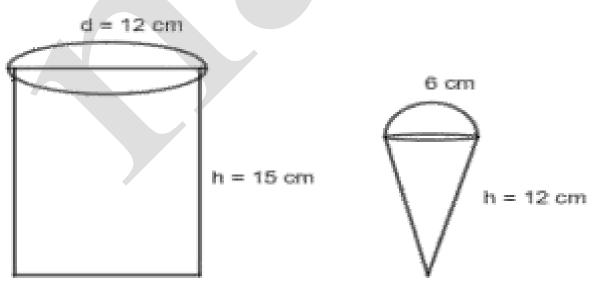
Diameter of the cylinder $d_1 = 12$ cm

Height of cone with hemispherical top $h_2 = 12$ cm

Diameter of cone with hemispherical top $d_2 = 6$ cm

To find,

The number of cones to be filled with ice cream



Hence Volume of the ice cream in the cylinder is to be filled in cones.

So,

Volume of ice cream in cylinder = $n \times$Volume of cones

Volume of ice cream in cylinder $= \pi r_1^2 h_1$

$= \pi \; 6^2 \quad 15$

Volume of ice cream cones = Volume of 1 ice cream cone + Volume of hemispherical shape on the top

$= n \times \frac{1}{3}\pi r_2^2 h_2 + \frac{2}{3}\pi r_2^3$

$= n \times \pi\left(\frac{1}{3} \times \; 3^{\,2} \times 12 + \left(\frac{2}{3} \times \; 3^{\,3}\right)\right)$

By equating both, we get,

$n = \dfrac{6^2 \times 15}{\frac{1}{3} \times 9 \times 12 + \frac{2}{3} \times \; 3^{\,2}}$

$n = \dfrac{36 \;\; 15 \quad 3}{108 \;\; 54}$

$n = 10$

$\therefore$ The number of ice cream cones that can be filled with the ice cream in the container is 10.

6. How many silver coins, 1.75 cmin diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions $5.5 \text{ cm} \times 10 \text{ cm} \times 3.5 \text{ cm}$?

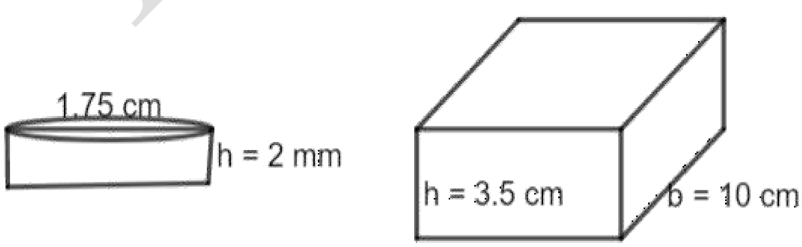
Ans: Diameter of the silver coin $= 1.75$ cm

Thickness of the silver coin $= 2$ mm

Dimension of a cuboid $= 5.5 \times 10 \times 3.5$

To find,

The number of silver coins melted to form the cuboid of above dimension



Radius of silver coin $= \frac{1.75}{2}$

$= 0.875 \text{ cm}$

The volume of the cuboid formed is equal to the number of coins melted to form it.

Volume of the cuboid = Volume of the n coins melted

$lbh = n \times \pi r^2 h_1$

$5.5 \times 10 \times 3.5 = n \times \pi \; 0.875^{2} \times 0.2$

$n = \frac{55 \times 10 \times 3.5}{0.875^{2} \times 0.2 \times \frac{22}{7}}$

$n = 400$

$\therefore$ The number of coins required to from a cuboid is 400.

7. A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm. Find the radius and slant height of the heap.

Ans: Given that,

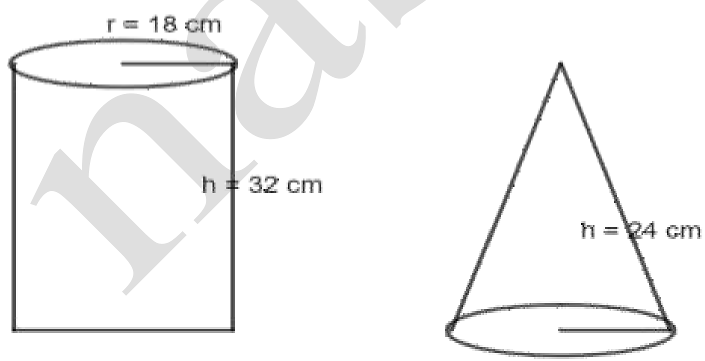
Height of the cylindrical bucket $h_1 = 32 \text{ cm}$

Radius of cylindrical bucket with circular end $r_1 = 18 \text{ cm}$

Height of conical heap formed $h_2 = 24 \text{ cm}$

To find,\

The radius and slant height of the heap



The volume of the sand in the cylindrical bucket is equal; to the volume of the sand that forms the conical heap.

So,

$$\pi r_1^2 h_1 = \frac{1}{3}\pi r_2^2 h_2$$

$$18^2 \times 32 = \frac{1}{3} \times r_2^2 \times 24$$

$$r_2^2 = \frac{3\times18\times18\times32}{24}$$

$$r_2^2 = 18^2 \times 2^2$$

$$r_2 = 36 \text{ cm}$$

Slant height, $l = \sqrt{r^2 + h^2}$

$$= \sqrt{36^2 + 24^2}$$

$$= 12\sqrt{13} \text{ cm}$$

$\therefore$ The radius of the conical heap is 36 cm and that of slant height is $12\sqrt{13}$ cm.

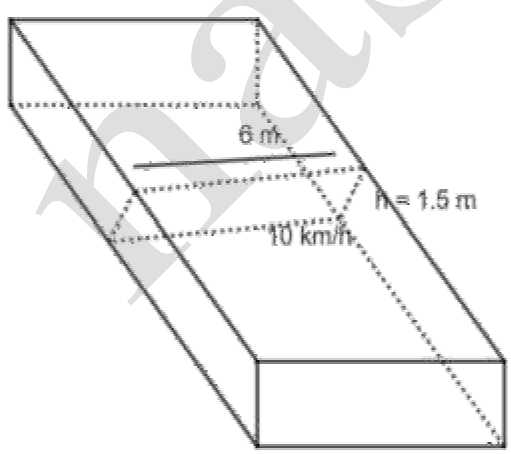
8. Water in canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/hr . How much area will it irrigate in 30 minutes, if 8 cm Is standing water needed?

Ans: Cross section of the canal $= 6 \text{ m} \times 1.5 \text{ m}$

Flowing with the speed of 10 km/h

To find,

The area it can irrigate in 30 minutes if the standing water needed is 8 cm



Area of cross section of canal $= 6 \times 1.5$

$= 9\ \text{m}^2$

Speed of the water $= 10$ km/h

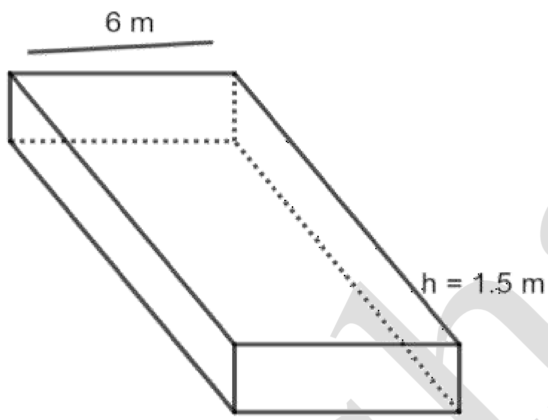
$= \frac{10000}{60}$ m/min

Volume of the water that flows in one minute from canal $= 9 \times \frac{10000}{60}$

$= 1500\ \text{m}^3$

Volume of the water that flows in 30 minute from canal $= 30 \times 1500$

$= 45000\ \text{m}^3$



Let the irrigated area be A. Volume of water irrigating the required area will be equal to the volume of water that flowed in 30 minutes from the canal.

Volume of the water flowing in 30 minutes from the canal = Volume of the water irrigating the required area

$$45000 = \frac{A \times 8}{100}$$

$$A = \frac{45000 \times 100}{8}$$

$$A = 562500\ \text{m}^2$$

$\therefore$ The area irrigated in 30 minutes is $562500\ \text{m}^2$

9. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2m in deep. If water flows through a pipe at the rate of 3 km/h , in how much time will the tank be filled?

Ans: Given That,

Diameter of pipe $d_1 = 20\text{ cm}$

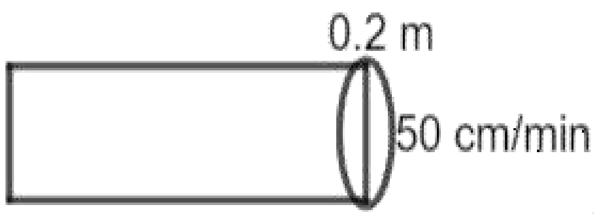
Diameter of cylindrical tank $d_2 = 10\text{ m}$

Height of the cylindrical tank $h_2 = 2\text{ m}$

Speed of water 30 km/h

To find,

The time at which the tank will be filled



Consider an area of cross section of the pipe as shown above:

Radius of pipe in m $r_1 = \dfrac{20}{2\times100}$

$= 0.1\text{ m}$

Radius of cylindrical tank $r_2 = \dfrac{10}{2}$

$= 5\text{ m}$

Area of cross section of the pipe $= \pi r_1^2$

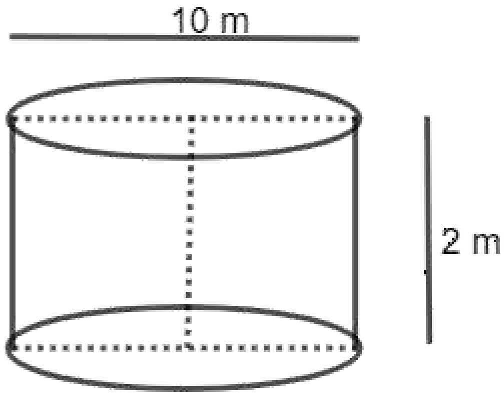
$= \pi \times 0.1^2$

$= 0.01\pi\text{ m}^2$

Speed of water $= 30$ km/h

$= \dfrac{3000}{60}$ m/min

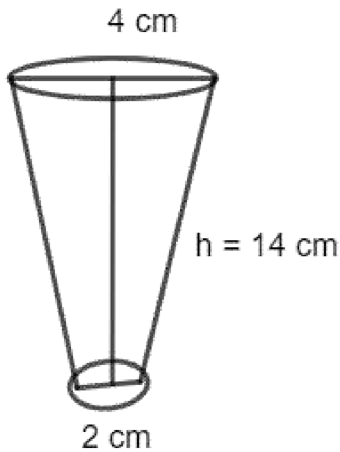
$= 50$ m/min



Volume of water flows in 1 min from the pipe $= 50 \times 0.01\pi$
$= 0.5\pi m^3$
Volume of water flows in t minute from the pipe $= t \times 0.5\pi$
Let us assume that the tank has been filled completely. Volume of water filled in t minutes in the tank is equal to the volume of water flowed out from the pipe in t minutes.
So,
$t \times 0.5\pi = \pi \times r_2^2 \times h_2$
$t = \frac{5^2 \times 2}{0.5}$
$t = 100$ minutes
$\therefore$ The tank will be filled in 100 minutes.

Exercise 13.4

1. A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass. [Use $\pi = \frac{22}{7}$]
Ans: Given that,
Height of the conical glass $h = 14\text{ cm}$
Diameter of upper base of circular part $d_1 = 4\text{ cm}$
Diameter of lower base of the circular part $d_2 = 2\text{ cm}$
To find,
The capacity of the glass



Radius of upper base of glass $r_1=\frac{4}{2}$

$=2$ cm

Radius of lower base of glass $r_2=\frac{2}{2}$

$=1$ cm

Volume of the conical glass $=\frac{1}{3}\pi h\ \ r_1^2+r_2^2+r_1r_2$

$=\frac{1}{3}\pi\times14\left[\ 2^{\ 2}+\ 1^{\ 2}+\ 2\times1\right]$

$=\frac{1}{3}\times\frac{22}{7}\times14\times7$

$=\frac{308}{3}$

$=102\frac{2}{3}$ cm^3

$\therefore$ The capacity of the drinking glass is $102\frac{2}{3}$ cm^3.

2. The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

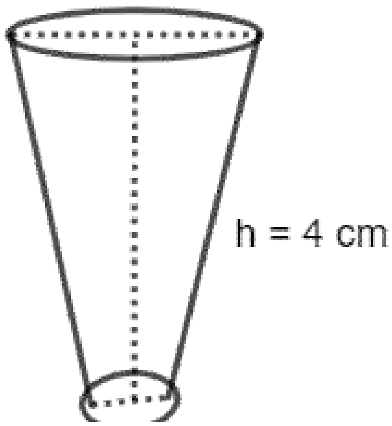
Ans: Slant height of the frustum $l=4$ cm

Perimeter of upper circular part $2\pi r_1=18$

Perimeter of lower circular part $2\pi r_2 = 6$

To find,

The curved surface area of the frustum



$2\pi r_1 = 18$

$r_1 = \frac{9}{\pi}$

$2\pi r_2 = 6$

$r_2 = \frac{6}{\pi}$

Curved surface area of frustum $= \pi\ r_1 + r_2\ l$

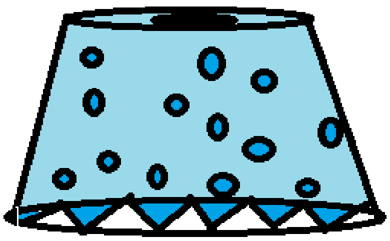
$$= \pi\left(\frac{9}{\pi} + \frac{3}{\pi}\right)4$$

$$= \pi \times \frac{48}{\pi}$$

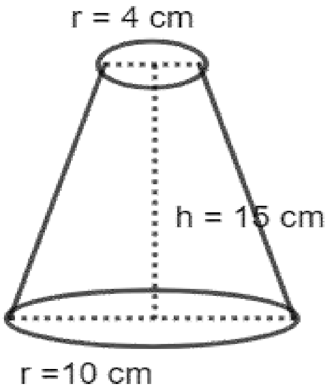
$$= 48\ \text{cm}^2$$

The curved surface area off the frustum is $48\ \text{cm}^2$.

3. A fez, the cap used by the Turks, is shaped like the frustum of a con (see the figure given below). If its radius on the open side is 10 cm , radius at the upper base is 4 cm and its slant height is 15 cm, find the area of the material used for making it. [Use $\pi = \frac{22}{7}$]



Ans: Radius of lower circular end of the frustum $r_1 = 10$ cm
Radius of upper circular end of the frustum $r_2 = 4$ cm
Slant height of the frustum $l = 15$ cm
To find,
The area of the material used for making it



Area of the material used = CSA of the frustum + Area of the upper circle
CSA of the frustum $= \pi\ r_1 + r_2\ l$

$= \pi\ 10 + 4\ 15$

$= \pi\ 14\ 15$

$= 210\pi$

Area of the upper circle $= \pi r_2^{\ 2}$

$= \pi\ 4^{\ 2}$

$= 16\pi$

Area of the material used $= 210\pi + 16\pi$

$= 226\pi$

$= 226 \times \frac{22}{7}$

$= 710\frac{2}{7}\ \text{cm}^2$

$\therefore$ The area of the material used for making the cap is $710\frac{2}{7}\ \text{cm}^2$.

4. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs.20 per liter. Also find the cost of metal sheet ,if it costs Rs.8 per 100 cm^2 . [Take $\pi = 3.14$]

Ans: Height of the frustum $h = 16$ cm

Radius of the upper circular end $r_1 = 20$ cm

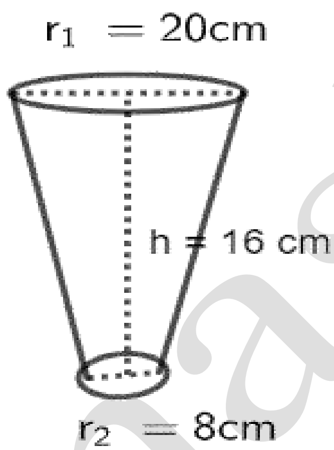
Radius of the lower circular end $r_2 = 8$ cm

Cost of milk per liter $=$ Rs.20

Cost of metal sheet per 100 $\text{cm}^3 =$ Rs.8

To find,

The cost of metal sheet that fill the container completely



Slant height of the frustum $l = \sqrt{r^2 + h^2}$

$= \sqrt{(r_1 - r_2)^2 + h^2}$

$= \sqrt{(20 - 8)^2 + (16)^2}$

$= \sqrt{(12)^2 + (16)^2}$

$= \sqrt{144 + 256}$
$= \sqrt{400}$
$= 20$ cm
The capacity of the container is equal to the volume of the frustum.
Volume of frustum $= \frac{1}{3}\pi h\left[r_1^2 + r_2^2 + r_1 r_2\right]$
$= \frac{1}{3}\pi \times 16\left[(20)^2 + (8)^2 + (20)(8)\right]$
$= \frac{1}{3}\pi \times 16(400 + 64 + 160)$
$= \frac{1}{3} \times 3.14 \times 16 \times 624$
$= 10,449.92 \text{ cm}^3$
We know that $1000 \text{ cm}^3 = 1 \text{ l}$
So, Volume of the frustum $= 10.45$liters
Cost of one liter milk $=$ Rs.20
Cost of 10.45 liter milk $= 10.45 \times 20$
$=$ Rs.209
Area of the metal sheet used to make the container $= \pi(r_1 + r_2)l + \pi r_2^2$
$= \pi(20 + 8)20 + \pi(8)^2$
$= 560\pi + 64\pi$
$= 624 \times 3.14$
Cost of 100 cm^2 sheet $=$ Rs.8
Cost of $624\pi \text{ cm}^2$ sheet $= 624 \times 3.14 \times \frac{8}{100}$
$=$ Rs.156.75
$\therefore$ The cost of the milk that can completely fill the container is Rs.209
The cost of the metal sheet that is used to make the container is Rs.156.75.

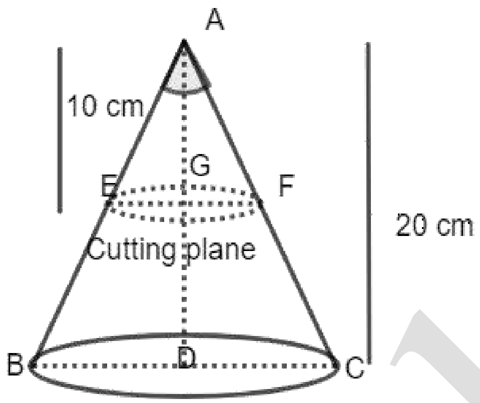
5. A metallic right circular cone 20 cm high and whose vertical angle is $60°$ is cut into two parts at the middle of its height by a plane parallel to its base. If

the frustum so obtained is drawn into a middle of a diameter $\frac{1}{16}$ cm, find the length of the wire. [Use $\pi = \frac{22}{7}$]

Ans: Given that,
Height of the cone $= 20$ cm

Diameter of the wire $= \frac{1}{16}$cm

To find,
The length of the wire



Consider $\triangle AEG$,

$\tan 30° = \frac{EG}{AG}$

$EG = \frac{10}{\sqrt{3}}$

$EG = \frac{10\sqrt{3}}{3}$ cm

Consider $\triangle ABD$,

$\tan 30° = \frac{BD}{AD}$

$BD = \frac{20}{\sqrt{3}}$

$BD = \frac{20\sqrt{3}}{3}$ cm

Thus the radius of the upper end of the frustum $r_1 = \frac{10\sqrt{3}}{3}$ cm

Radius of the lower end of the container $r_2 = \frac{20\sqrt{3}}{3}$ cm

Height of the container $h = 10$ cm

Volume of the frustum $= \frac{1}{3}\pi h\ r_1^2 + r_2^2 + r_1r_2$

$$= \frac{1}{3}\pi \times 10\left[\left(\frac{10\sqrt{3}}{3}\right)^2 + \left(\frac{20\sqrt{3}}{3}\right)^2 + \left(\frac{10\sqrt{3}\times 20\sqrt{3}}{3\times 3}\right)\right]$$

$$= \frac{10}{3}\pi\left[\frac{100}{3} + \frac{400}{3} + \frac{200}{3}\right]$$

$$= \frac{10}{3}\times\frac{22}{7}\times\frac{700}{3}$$

$$= \frac{22{,}000}{9}\ \text{cm}^3$$

Volume of the frustum is $\frac{22{,}000}{9}$ cm^3

Radius of the wire $r = \frac{1}{16}\times\frac{1}{2}$

$$= \frac{1}{32}\text{cm}$$

Let us consider the length of the wire to be l.

Volume of the wire = Area of the cross section $\times$ Length

$$= \pi r^2 \times l$$

$$= \pi\left(\frac{1}{32}\right)^2 l$$

Volume of the frustum = Volume of the wire

$$\frac{22000}{9} = \frac{22}{7}\times\frac{1}{1024}\times l$$

$l = 796444.44$ cm

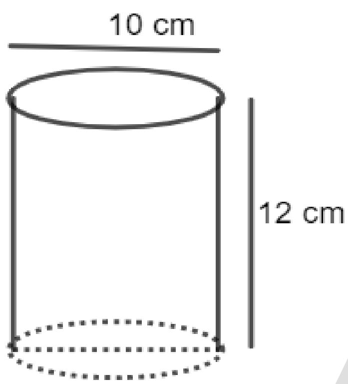
$l = 7964.44$ m

$\therefore$ The length of the wire is 7964.44 m.

Exercise 13.5

1. A copper wire 3 mm in diameter, is wound about a cylinder whose length is 12 cm , diameter 10 cm , so as to cover the curved surface area of the cylinder. Find the length and mass of the wire, assuming the density of copper to be 8.88 g per cm^3.

Ans: Diameter of the copper wire $= 3\text{ mm}$
Length of the cylinder $= 12\text{ cm}$
Diameter of the cylinder $= 10\text{ cm}$
Density of the copper per $\text{cm}^3 = 8.88\text{g}$



We have to note that a round of wire can cover the cylinder of height 3 mm.
Length of the wire in a round = Circumference of the base cylinder
$= 2\pi r$
$= 2\pi\ 5$
$= 10\pi$

Number of rounds $= \dfrac{\text{height of cylinder}}{\text{Diameter of the wire}}$

$= \dfrac{12}{0.3}$

$= 40\text{ rounds}$

Length of the wire in 40 rounds $= 40 \times 10\pi$

$= 400 \times \dfrac{22}{7}$

$= 1257.14\text{ cm}$
$= 12.57\text{ m}$

Radius of the wire $= \frac{0.3}{2}$

$= 0.15$

Volume of the wire = Area of the cross section $\times$ Length of the wire

$= \pi \; 0.15^{2} \times 1257.14$

$= 88.898 \text{ cm}^3$

Mass of the wire = Volume $\times$ density

$= 88.898 \times 8.88$

$= 789.41 \text{ g}$

$\therefore$ The length of the wire is 12.57 m

The mass of the wire is 789.41 g

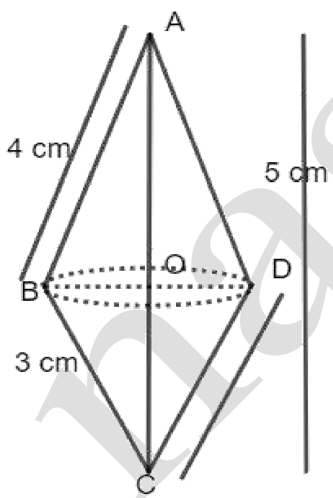
2. A right triangle whose sides are 3 cm and 4 cm (other than hypotenuse) are made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Choose value of π as found appropriate).

Ans: Given that,

The sides of a triangle are **3,4.**

To find,

The volume and surface area of the double cone.



The double cone formed by rotating the right triangle about its hypotenuse is shown in the above figure.

By Pythagorean theorem,

$AC = \sqrt{3^2 + 4^2}$

$= \sqrt{25}$

$= 5 \text{ cm}$

Area of $\triangle ABC = \frac{1}{2} \times AB \times AC$

$\frac{1}{2} \times AC \times OB = \frac{1}{2} \; 4 \;\; 3$

$\frac{5}{2} OB = 6$

$OB = \frac{12}{5}$

$OB = 2.4 \text{ cm}$

Volume of double cone = Volume of cone 1+ Volume of cone 2

$= \frac{1}{3}\pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$

$= \frac{1}{3}\pi r^2 \;\; h_1 + h_2$

$= \frac{1}{3} \times 3.14 \times \;\; 2.4^{\;2} \;\; OA + AC$

$= \frac{1}{3} \times 3.14 \times \;\; 2.4^{\;2} \, 5$

$= 30.14 \text{ cm}^3$

Surface area of the double cone $= \pi r l_1 + \pi r l_2$

$= \pi r \;\; l_1 + l_2$

$= \pi r \;\; 4 + 3$

$= 3.14 \times 2.4 \times 7$

$= 52.75 \text{ cm}^2$

$\therefore$ The volume of the double cone is 30.14 cm^3 .

The surface area of the double cone is 52.75 cm^2

3. A cistern, internally measuring 150 cm $\times$ 120 cm $\times$ 110 cm, has 129600 cm^3 of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one-seventeenth of its own volume of water. How many bricks can be put in without overflowing the water, each brick being 22.5 cm $\times$ 7.5 cm $\times$ 6.5 cm?

Ans: Given that,

Dimension of cistern $=150\times120\times110$

Volume of water in cistern $=129600\ \text{cm}^3$

Dimension of brick $=22.5\times7.5\times6.5$

To find,

The number of bricks to be put in the cistern without the overflow of water

Volume of cistern $=150\times120\times110$

$=1980000\ \text{cm}^3$

Volume of bricks to be filled in cistern $=1980000-129600$

$=1850400\ \text{cm}^3$

Let n be the number of bricks that is to be placed in the cistern.

Volume of n bricks $=n\times22.5\times7.5\times6.5$

$=1096.875n$

As each brick absorbs one-seventeenth of its volume, therefore, volume absorbed by these bricks are,

$$=\left(\frac{n}{17}\right)1096.875$$

$$1850400+\left(\frac{n}{17}\right)1096.875=1096.875n$$

$$\frac{16n}{17}1096.875=1850400$$

$n=1792.41$

$\therefore 1792$ bricks can be placed in a cistern without the overflow of water.

4. In One fortnight of the given month, there was a rainfall of $10\ \text{cm}$ in a river valley. If the area of the valley is $97280\ \text{km}^2$, show that the total rainfall was approximately equivalent to the addition to the normal water of the three rivers each $1072\ \text{km}$ long, $75\ \text{m}$ wide and $3\ \text{m}$ deep.

Ans: Given that,

Area of the valley $=97280\ \text{km}^2$

Amount of rise in water level in valley $h=10\ \text{cm}$

Dimension of rivers $=1072\times75\times3$

To show that the total rainfall is approximately equivalent to the addition to normal water of three rivers.

Amount of rise in the water $h=10\ \text{cm}$

$= \frac{10}{100000}$ km

$= \frac{1}{10000}$ km

Amount of rainfall in 14 days $= A \times h$

$= 97820 \times \frac{1}{10000}$

$= 9.828 \text{ km}^3$

Amount of rainfall in a day $= \frac{9.828}{14}$

$= 0.702 \text{ km}^3$

Volume of water in three rivers $= 3lbh$

$= 3\ 1072 \text{ km} \times 75 \text{ m} \times 3 \text{ m}$

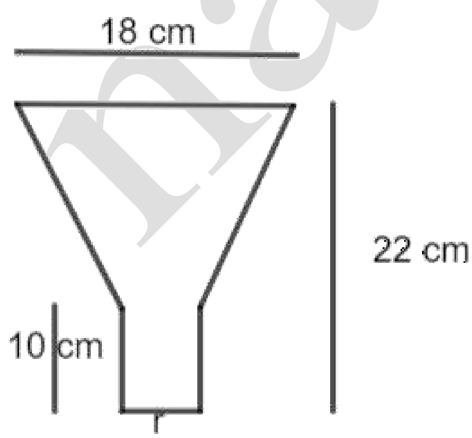
$= 3\left(1072 \text{ km} \times \frac{75}{1000} \text{ km} \times \frac{3}{1000} \text{ km}\right)$

$= 3\ 0.2412$

$= 0.7236 \text{ km}^3$

This shows that the rainfall is approximately equal to the amount of water in three rivers.

5. An oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attached to a frustum of a cone. If the total height is 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm , find the area of the tin sheet required to make the funnel(see the given figure).



Ans: Height of a cylindrical part $h_1 = 10$ cm

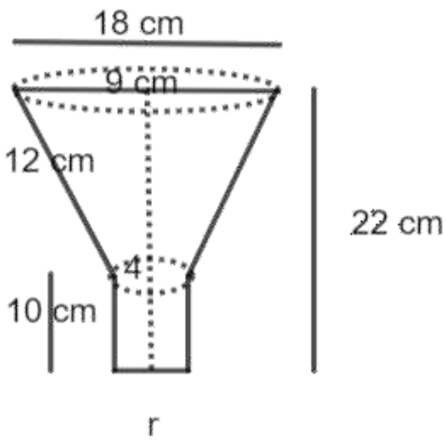
Total height of the funnel $h = 22$ cm

Diameter of the circular portion in conical part$d_2 = 18$ cm

Diameter of the circular portion in cylindrical part $d_1 = 8$ cm

To find,

The area of the tin sheet required to make the funnel



Total height of the frustum $h_2 = 22 - 10$

$= 12$ cm

Slant height of the frustum $l = \sqrt{(r_2 - r_1)^2 + h^2}$

$= \sqrt{(9-4)^2 + (12)^2}$

$= \sqrt{169}$

$= 13$ cm

Area of the tin sheet required = CSA of frustum part +CSA of cylindrical part

$= \pi(r_2 + r_1)l + 2\pi r_1 h_1$

$= \left(\frac{22}{7} \times 169\right) + \left(\frac{22}{7} \times 2 \times 4 \times 10\right)$

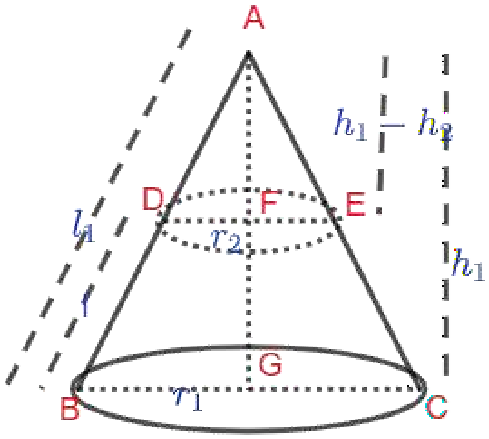
$= \frac{22}{7}(169 + 80)$

$= 782\frac{4}{7}\ \text{cm}^2$

Area of the tin sheet required to make the funnel is $782\frac{4}{7}\ \text{cm}^2$.

6. Derive the formula for the curved surface area and total surface area of the frustum of the cone.

Ans: To derive the formula for CSA and TSA of the cone



Let ABC be a cone. A frustum $DECB$is cut by a plane parallel to its base. Let r_1, r_2 be the radii of the ends of the frustum of the cone and h be the height of the frustum of the cone.

Consider $\triangle ABG$ and $\triangle ADF$,

$DF \parallel BG$

$\therefore \triangle ABG \sim \triangle ADF$

$$\frac{DF}{BG} = \frac{AF}{AG} = \frac{AD}{AB}$$

$$\frac{r_2}{r_1} = \frac{h_1 - h}{h_1} = \frac{l_1 - l}{l_1}$$

$$\frac{r_2}{r_1} = 1 - \frac{h}{h_1} = 1 - \frac{l}{l_1}$$

$$1 - \frac{l}{l_1} = \frac{r_2}{r_1}$$

$$\frac{l}{l_1} = 1 - \frac{r_2}{r_1}$$

$$= \frac{r_1 - r_2}{r_1}$$

By taking reciprocal and solving for l_1, we get,

$$l_1 = l\frac{r_1}{r_1 - r_2}$$

CSA of frustum DECB = CSA of con $ABC - $CSA of cone ADE

$$= \pi r_1 l_1 - \pi r_2 \ l_1 - l$$

$$= \pi r_1\left(\frac{lr_1}{r_1 - r_2}\right) - \pi r_2\left(\frac{lr_1}{r_1 - r_2} - l\right)$$

On solving this, we get,

$$= \frac{\pi r_1^2 l}{r_1 - r_2} - \frac{\pi r_2^2 l}{r_1 - r_2}$$

Taking πl as common, we get,

$$= \pi l\left[\frac{r_1^2 - r_2^2}{r_1 - r_2}\right]$$

CSA of frustum $= \pi \ r_1 + r_2 \ l$

Total surface area of frustum = CSA of frustum + Area of upper circular ends + area of lower circular end

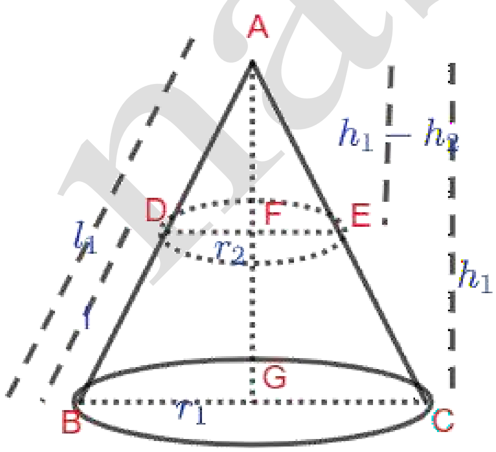
$$= \pi l \ r_1 + r_2 \ + \pi r_1^2 + \pi r_2^2$$

$$= \pi \ l \ r_1 + r_2 \ + r_1^2 + r_2^2$$

Hence the formula for the curved surface area and the total surface area of the frustum has been derived.

7. Derive the formula for the volume of the frustum of the cone.

Ans: To derive the formula for the volume of the frustum of a cone.



Let ABC be a cone. A frustum DECBis cut by a plane parallel to its base. Let r_1, r_2 be the radii of the ends of the frustum of the cone and h be the height of the frustum of the cone.

Consider $\triangle ABG$ and $\triangle ADF$,

$DF \parallel BG$

$\therefore \triangle ABG \sim \triangle ADF$

$$\frac{DF}{BG} = \frac{AF}{AG} = \frac{AD}{AB}$$

$$\frac{r_2}{r_1} = \frac{h_1 - h}{h_1} = \frac{l_1 - l}{l_1}$$

$$\frac{r_2}{r_1} = 1 - \frac{h}{h_1} = 1 - \frac{l}{l_1}$$

$$1 - \frac{h}{h_1} = \frac{r_2}{r_1}$$

$$\frac{h}{h_1} = 1 - \frac{r_2}{r_1}$$

$$= \frac{r_1 - r_2}{r_1}$$

$$h_1 = h\frac{r_1}{r_1 - r_2}$$

Volume of the frustum of the cone = Volume of cone ABC - Volume of the cone ADE

$$= \frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 \;\; h_1 - h$$

$$= \frac{\pi}{3}\left[r_1^2 h_1 - r_2^2 \;\; h_1 - h \;\right]$$

Substitute $h_1 = h\frac{r_1}{r_1 - r_2}$

$$= \frac{\pi}{3}\left[r_1^2 \left(\frac{hr_1}{r_1 - r_2} \right) - r_2^2 \left(\frac{hr_1}{r_1 - r_1} - h \right) \right]$$

$$\frac{}{3} \; \frac{hr_1^3}{r_1 \quad r_2} \quad \frac{hr_2^3}{r_1 \quad r_2}$$

$$= {}^{\pi}\left[\; \_ \quad - \quad \_ \; \right]$$

$$= \frac{\pi}{3} h \left[\frac{r_1 - r_2}{r_1 - r_2} \; r_1^2 + r_2^2 + r_1 r_2 \; \right]$$

$$= \frac{1}{3} \pi h \; r_1^2 + r_2^2 + r_1 r_2$$

Thus the formula for the volume of the cone has been derived.