Class 10 Maths NCERT Solutions

Chapter 12 – Areas related to circles

Exercise 12.1

1. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Ans:

Given that,

Radius of 1st circle = $r_1 = 19$ cm

Radius of 2nd circle = $r_2 = 9$ cm

Circumference of 3rd circle = Circumference of 1st circle + Circumference of 2nd circle

Let the radius of 3rd circle be r.

Now,

Circumference of 1st circle = $2\pi r_1$

$= 2\pi(19)$

$= 38\pi$

Circumference of 2nd circle = $2\pi r_2$

$= 2\pi(9)$

$= 18\pi$

Circumference of 3rd circle = $2\pi r$

Using given condition,

$$2\pi r = 38\pi + 18\pi$$
$$= 56\pi$$
$$\Rightarrow r = \frac{56\pi}{2\pi}$$
$$= 28.$$

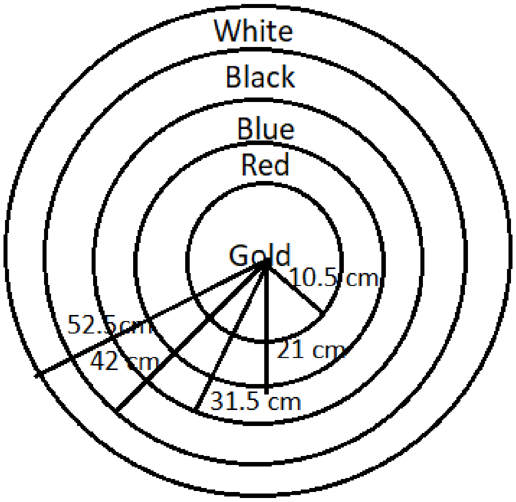
Therefore, the radius of the circle which having circumference equal to the sum of the circumference of the given two circles is 28 cm.

2. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

Ans:
Given that,
Radius of 1st circle = $r_1 = 8$ cm
Radius of 2nd circle = $r_2 = 6$ cm
Area of 3rd circle = Area of 1st circle + Area of 2nd circle
Let the radius of 3rd circle be r.
Area of 1st circle = πr_1^2
$= \pi(8)^2$
$= 64\pi$
Area of 2nd circle = πr_2
$= \pi r_2^2$
$= 36\pi$
Using given condition,
$\pi r^2 = \pi r_1^2 + \pi r_2^2$
$= 64\pi + 36\pi$
$= 100\pi$
$\Rightarrow r^2 = 100$
$r = \pm 10$
We know that, the radius cannot be negative. Therefore, the radius of the circle having area equal to the sum of the areas of the other two circles, is 10 cm.

3. Given figure depicts an archery target marked with its five scoring areas from the center outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions. [Use $\pi = \frac{22}{7}$]

Ans:



Given that,

Radius of gold region (i.e., 1st circle) = r_1

$$= \frac{21}{2}$$

$$=10.5.$$

Each circle is 10.5 cm wider than the previous circle.

Thus, radius of 2^{nd} circle = r_2

$= 10.5+10.5$

$= 21$ cm

Radius of 3^{rd} circle = r_3

$= 21 + 10.5$

$= 31.5$ cm

Radius of 4^{th} circle = r_4

$= 31.5 + 10.5$

$= 42$ cm

Radius of 5^{th} circle = r_5

$= 42 + 10.5$

$= 52.5$ cm

According to given condition,

Area of gold region = Area of 1st circle

$\Rightarrow \pi r_1^2 = \pi(10.5)^2$

$= 346.5 cm^2$

Area of red region = Area of 2nd circle − Area of 1st circle

$$= \pi r_2^2 - \pi r_1^2$$
$$= \pi(21)^2 - \pi(10.5)^2$$
$$= 441\pi - 110.25\pi$$
$$= 330.75\pi$$
$$= 1039.5 cm^2$$

Area of blue region = Area of 3rd circle − Area of 2nd circle

$$= \pi r_3^2 - \pi r_2^2$$
$$= \pi(31.5)^2 - \pi(21)^2$$
$$= 992.25\pi - 441\pi$$
$$= 551.25\pi$$
$$= 1732.5 cm^2$$

Area of black region = Area of 4th circle − Area of 3rd circle

$$= \pi r_4^2 - \pi r_3^2$$
$$= \pi(42)^2 - \pi(31.5)^2$$
$$= 1764\pi - 992.25\pi$$
$$= 771.75\pi$$
$$= 2425.5 cm^2$$

Area of white region = Area of 5th circle − Area of 4th circle

$$= \pi r_5^2 - \pi r_4^2$$
$$= \pi(52.5)^2 - \pi(42)^2$$
$$= 2756.25\pi - 1764\pi$$
$$= 992.25\pi$$
$$= 3118.5 cm^2$$

Therefore, areas of gold, red, blue, black, and white regions are $346.5 cm^2$, $1039.5 cm^2$, $1732.5 cm^2$, $2425.5 cm^2$ and $3118.5 cm^2$ respectively.

4. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is traveling at a speed of 66 km per hour?

Ans:

Given that,

Diameter of the wheel of the car = 80 cm

Thus, radius of the wheel of the car = $r = 40$ cm

Speed of car = 66 km/hour

We know that,

Circumference of wheel = $2\pi r$

$= 2\pi(40)$

$= 80\pi cm$

Speed of car $= \frac{66 \times 100000}{60} cm / min$

$= 1,10,000 cm / min$

Now, distance travelled by the car in 10 minutes $= 110000 \times 10$

$= 11,00,000 cm$

Let the number of revolutions of the wheel of the car be n.

We know that,

Distance travelled in 10 minutes = n × Distance travelled in 1 revolution (i.e., circumference)

$\Rightarrow 1100000 = n \times 80\pi$

$= \frac{35000}{8}$

$= 4375$

Therefore, each wheel of the car will make 4375 revolutions.

5. Tick the correct answer in the following and justify your choice: If the perimeter and the area of a circle are numerically equal, then the radius of the circle is

(A) 2 units (B) π units (C) 4 units (D) 7 units

Ans:

Given that,

the circumference and the area of the circle are equal.

Let the radius (to be found) of the circle be r

Thus,

Circumference of circle $= 2\pi r$ and

Area of circle $= \pi r^2$

According to given condition,

$2\pi r = \pi r^2$

$\Rightarrow 2 = r$

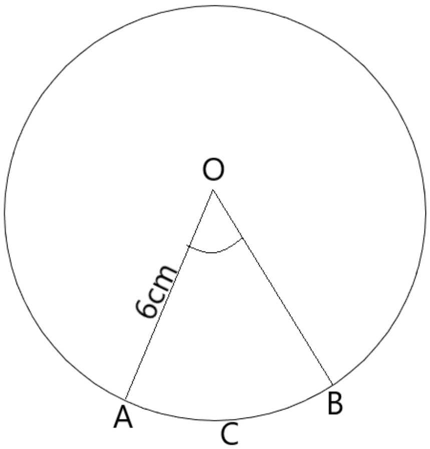
Therefore, the radius of the circle is 2 units.

Hence, the correct answer is A.

Exercise 12.2

1. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°. [Use $\pi = \frac{22}{7}$]

Ans:



Given that,

Radius of the circle = $r = 6\text{cm}$

Angle made by the sector with the center, $\theta = 60^\circ$

Let OACB be the sector of the circle making 60° angle at center O of the circle.

We know that area of sector of angle $= \frac{\theta}{360^\circ} \times \pi r^2$

Thus, Area of sector OACB $= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times (6)^2$

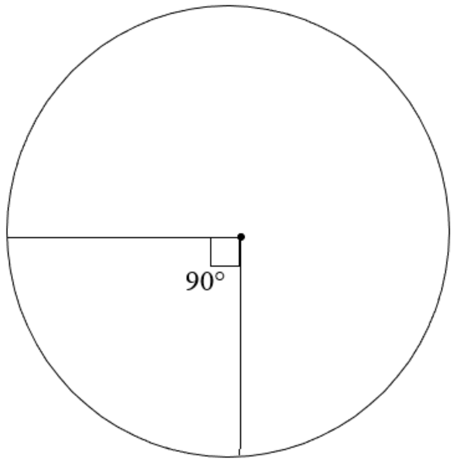
$$= \frac{1}{6} \times \frac{22}{7} \times 6 \times 6$$

$$= \frac{132}{7}\text{cm}^2$$

Therefore, the area of the sector of the circle making 60° at the center of the circle is $\frac{132}{7}\text{cm}^2$.

2. Find the area of a quadrant of a circle whose circumference is 22 cm. [Use $\pi = \frac{22}{7}$]

Ans:



Given that,
Circumference = 22 cm
Let the radius of the circle be r .
According to the given condition,

$$2\pi r = 22$$
$$\Rightarrow r = \frac{22}{2\pi}$$
$$= \frac{11}{\pi}$$

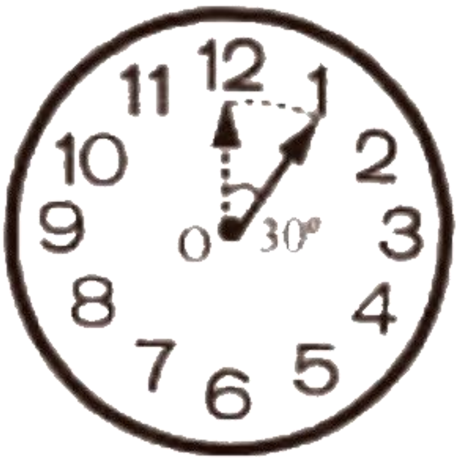
We know that, quadrant of circle subtends 90° angle at the center of the circle.
Thus,

$$\text{Area of such quadrant of the circle } = \frac{90^\circ}{360^\circ} \times \pi \times r^2$$
$$= \frac{1}{4} \times \pi \times \left(\frac{11}{\pi}\right)^2$$
$$= \frac{121}{4\pi}$$
$$= \frac{77}{8} \text{cm}^2$$

Hence, the area of a quadrant of a circle whose circumference is 22 cm is $= \frac{77}{8} \text{cm}^2$.

3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes. [Use $\pi = \frac{22}{7}$]

Ans:



Given that,

Radius of clock or circle = r = 14 cm.

We know that, in 1 hour (i.e., 60 minutes), the minute hand rotates 360°.

Thus, in 5 minutes, minute hand will rotate $= \frac{360^\circ}{60^\circ} \times 5$

$= 30^\circ$

Now,

the area swept by the minute hand in 5 minutes = the area of a sector of 30° in a circle of 14 cm radius.

Area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

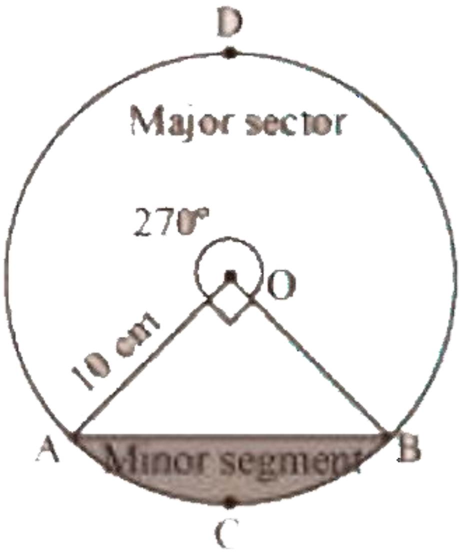
Thus, Area of sector of $30^\circ = \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14$

$= \frac{11 \times 14}{3}$

$= \frac{154}{3} \text{cm}^2$

Therefore, the area swept by the minute hand in 5 minutes is $\frac{154}{3} \text{cm}^2$.

4. A chord of a circle of radius 10 cm subtends a right angle at the center. Find the area of the corresponding:
[Use $\pi = 3.14$]
Ans:



Given that,
Radius of the circle $= r = 10\text{cm}$
Angle subtended by the cord = angle for minor sector$= 90^\circ$
Angle for minor sector $= 360^\circ - 90^\circ = 270^\circ$
(i) Minor segment
Ans: It is evident from the figure that,

Area of minor segment ACBA = Area of minor sector OACB − Area of ΔOAB

Thus,

$$\text{Area of minor sector OACB } = \frac{90^\circ}{360^\circ} \times \pi r^2 \;\; = \frac{1}{4} \times 3.14 \times (10)^2 \;\; = 78.5\text{cm}^2$$

$$\text{Area of } \Delta\text{OAB } = \frac{1}{2} \times \text{OA} \times \text{OB} = \frac{1}{2} \times (10)^2 = 50\text{cm}^2$$

Area of minor segment ACBA $= 78.5 - 50 = 28.5\text{cm}^2$

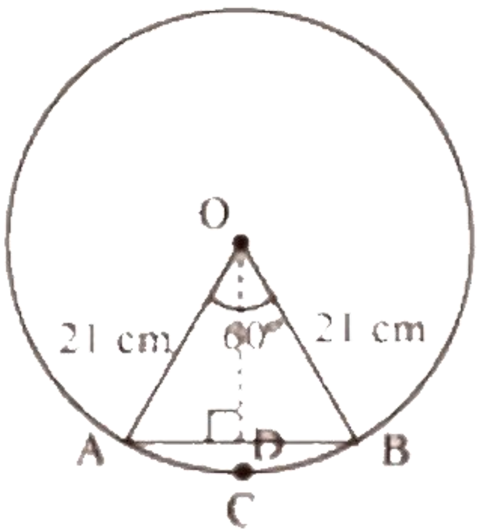
Hence, area of minor segment is 28.5cm^2

(ii) Major sector
Ans: It is evident from the figure that,

$$\text{Area of major sector OADB } = \frac{270^\circ}{360^\circ} = \frac{3}{4} \times 3.14 \times (10)^2 = 235.5\text{cm}^2.$$

Hence, area of major sector is 235.5cm^2.

5. In a circle of radius 21 cm, an arc subtends an angle of 60° at the center.
[Use $\pi = \frac{22}{7}$]
Find:
Ans:



Given that,
Radius of circle = r = 21 cm
Angle subtended by the given arc = $\theta = 60^\circ$
(i) The length of the arc
Ans: We know that, Length of an arc of a sector of angle $\theta = \frac{\theta}{360^\circ} \times 2\pi r$

Thus, Length of arc ACB $= \frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21$

$= 22\text{cm}$

Hence, length of the arc of given circle is 22cm.

(ii) Area of the sector formed by the arc

Ans: We know that, Area of sector OACB $= \frac{60^\circ}{360^\circ} \times \pi r^2$

$= 231\text{cm}^2$

Hence, area of the sector formed by the arc of the given circle is 231cm^2.

(iii) Area of the segment formed by the corresponding chord

Ans: In $\triangle OAB$,

As radius $OA = OB$

$\Rightarrow \angle OAB = \angle OBA$

$\angle OAB + \angle AOB + \angle OBA = 180^\circ$

$2\angle OAB + 60^\circ = 180^\circ$

$\angle OAB = 60^\circ$

Therefore, $\triangle OAB$ is an equilateral triangle.

Now, area of $\triangle OAB = \frac{\sqrt{3}}{4} \times (\text{side})^2$

$$= \frac{\sqrt{3}}{4} \times (r)^2$$

$$= \frac{\sqrt{3}}{4} \times (21)^2$$

$$= \frac{441\sqrt{3}}{4} \text{cm}^2$$

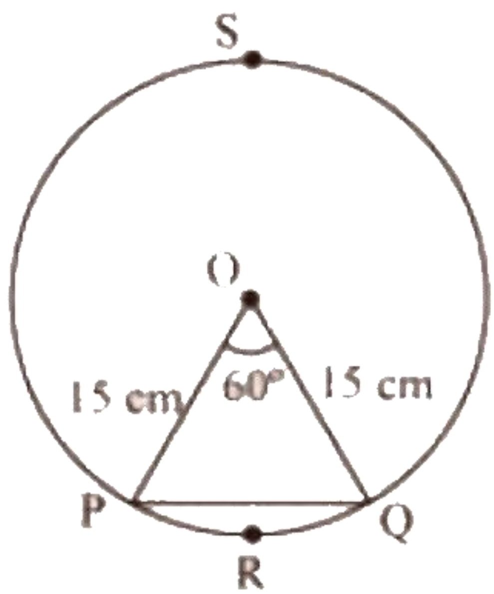
We know that, Area of segment ACB = Area of sector OACB − Area of $\triangle OAB$

$$= \left(231 - \frac{441\sqrt{3}}{4}\right) \text{cm}^2.$$

Hence, Area of the segment formed by the corresponding chord in circle is $\left(231 - \frac{441\sqrt{3}}{4}\right) \text{cm}^2$.

6. A chord of a circle of radius 15 cm subtends an angle of 60° at the center. Find the areas of the corresponding minor and major segments of the circle. [Use $\pi = 3.14$ and $\sqrt{3} = 1.73$]

Ans:



Given that,

Radius of circle = r = 15 cm

Angle subtended by chord $= \theta = 60^\circ$

Area of circle $= \pi r^2 = 3.14(15)^2$

$= 706.5 cm^2$

Area of sector OPRQ $= \frac{60^\circ}{360^\circ} \times \pi r^2$

$= \frac{1}{6} \times 3.14(15)^2 = 117.75 cm^2$

Now, for the area of major and minor segments,

In $\triangle OPQ$,

Since, $OP = OQ$

$\Rightarrow \angle OPQ = \angle OQP$

$\angle OPQ = 60^\circ$

Thus, $\triangle OPQ$ is an equilateral triangle.

Area of $\triangle OPQ$ $= \frac{\sqrt{3}}{4} \times (side)^2$

$= \frac{\sqrt{3}}{4} \times (r)^2$

$= \frac{225\sqrt{3}}{4} = 97.3125 cm^2$.

Now,

Area of minor segment PRQP = Area of sector OPRQ − Area of $\triangle OPQ$

$= 117.75 - 97.3125$

$= 20.4375 cm^2$

Area of major segment PSQP = Area of circle − Area of minor segment PRQP

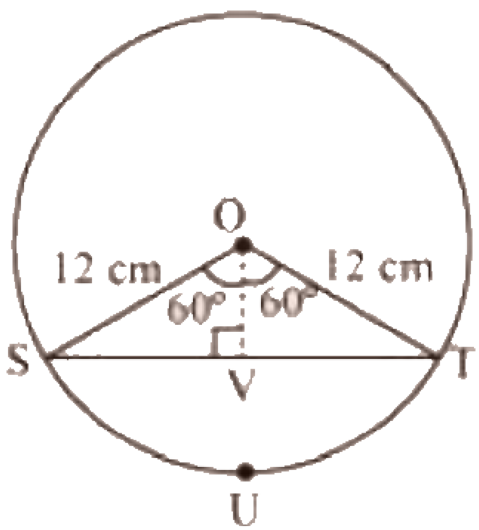
$= 706.5 - 20.4375$

$= 686.0625 cm^2$

Therefore, the areas of the corresponding minor and major segments of the circle are $20.4375 cm^2$ and $686.0625 cm^2$ respectively.

7. A chord of a circle of radius 12 cm subtends an angle of 120° at the center. Find the area of the corresponding segment of the circle. [Use $\pi = 3.14$ and $\sqrt{3} = 1.73$]

Ans:



Drawing a perpendicular OV on chord ST bisecting the chord ST such that SV=VT

Now, values of OV and ST are to be found.

Therefore,

In $\triangle OVS$,

$$\cos 60^\circ = \frac{OV}{OS}$$

$$\Rightarrow \frac{OV}{12} = \frac{1}{2}$$

$$\Rightarrow OV = 6 cm$$

Also, $\frac{SV}{SO} = \sin 60^\circ$

$\Rightarrow \frac{SV}{12} = \frac{\sqrt{3}}{2}$

$\Rightarrow SV = 6\sqrt{3}$

Now, $ST = 2SV$

$= 2 \times 6\sqrt{3} = 12\sqrt{3}\text{cm}$

Area of $\triangle OST = \frac{1}{2} \times ST \times OV$

$= \frac{1}{2} \times 12\sqrt{3} \times 6$

$= 62.28\text{cm}^2$

Area of sector OSUT $= \frac{120^\circ}{360^\circ} \times \pi(12)^2$

$= 150.42\text{cm}^2$

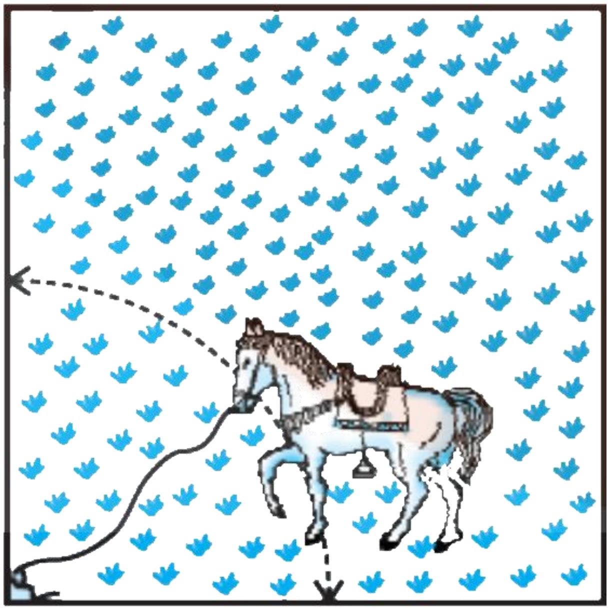
Area of segment SUTS = Area of sector OSUT − Area of $\triangle OVS$

$= 150.72 - 62.28$

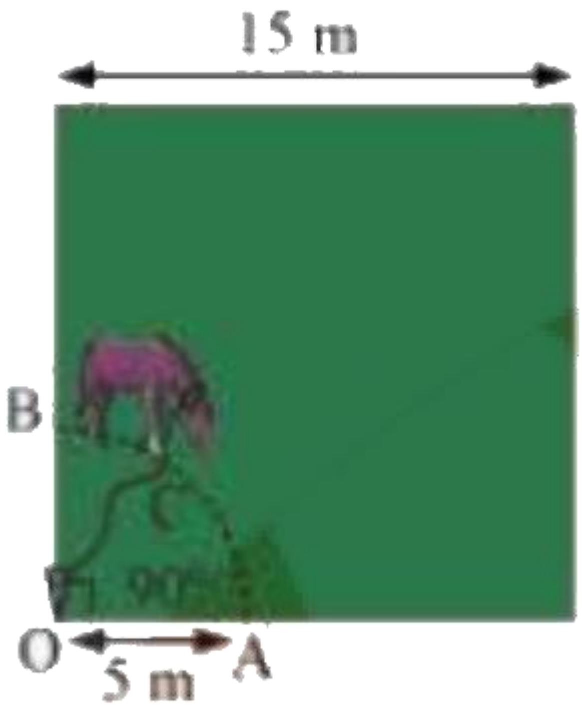
$= 88.44\text{cm}^2$

Hence, the area of the corresponding segment of the circle is 88.44cm^2.

8. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see the given figure). [Use $\pi = 3.14$]



Find
Ans:



From the above figure, it is clear that the horse can graze a sector of 90° in a circle of 5 m radius.
Hence,
$\theta = 90^\circ$
$r = 5m$

(i) The area of that part of the field in which the horse can graze.
Ans: It is evident from the figure,
Area that can be grazed by horse = Area of sector OACB

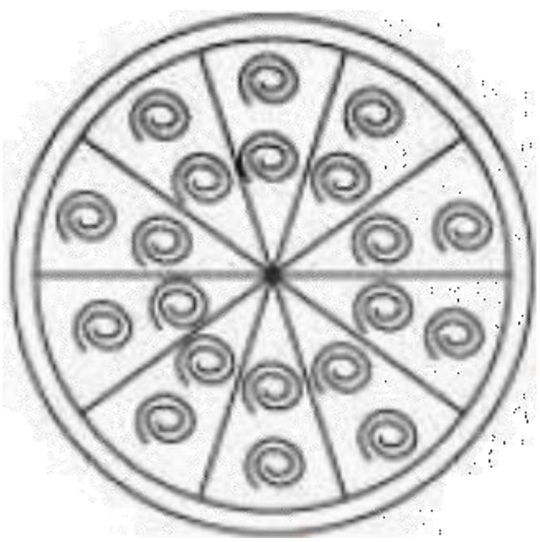
$$= \frac{90^\circ}{360^\circ}\pi r^2$$
$$= \frac{1}{4} \times 3.14 \times (5)^2 = 19.625 m^2$$

(ii) The increase in the grazing area if the rope were 10 m long instead of 5 m.
Ans: It is evident from the figure,
Area that can be grazed by the horse when length of rope is 10 m long

$$= \frac{90^\circ}{360^\circ} \times \pi \times (10)^2$$
$$= 78.5 m^2$$

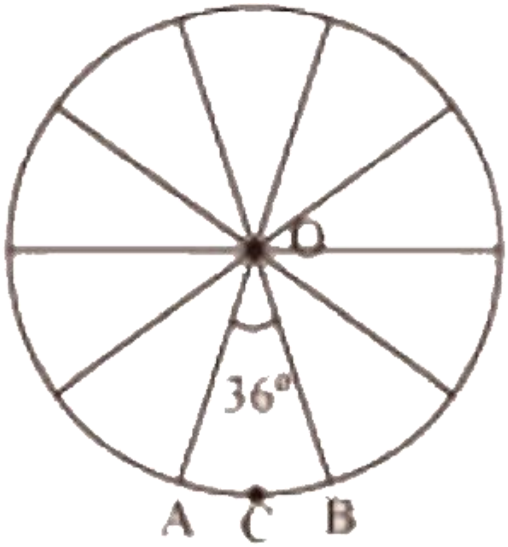
Therefore, the increase in grazing area for horse $= (78.5 - 19.625) m^2 = 58.875 m^2$.

9. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure. [Use $\pi = \frac{22}{7}$]



Find:
Ans:
Given that,

Radius of the circle $= r = \frac{\text{diameter}}{2} = \frac{35}{2}$mm



It can be observed from the figure that each of 10 sectors of the circle is subtending 36° (i.e., 360°/10=36°) at the center of the circle.

(i) The total length of the silver wire required.

Answer: It is evident from the figure that,

Total length of wire required will be the length of 5 diameters and the circumference of the brooch.

Circumference of brooch $= 2\pi r$

$$= 2 \times \frac{22}{7} \times \left(\frac{35}{2}\right)$$

$$= 110\text{mm}$$

Length of wire required $= 110 + (5 \times 35)$

$$= 285\text{mm}$$

Therefore, The total length of the silver wire required is 285mm.

(ii) The area of each sector of the brooch.

Answer: It is evident from the figure that,

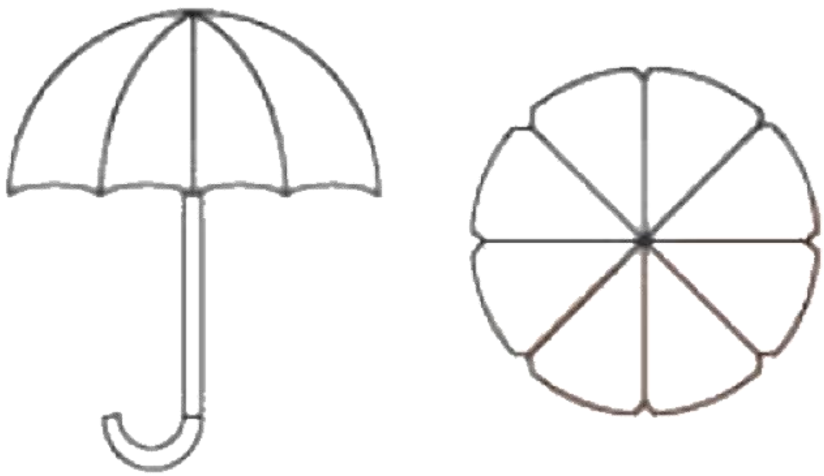
Area of each sector $= \frac{36^\circ}{360^\circ} \times \pi r^2$

$$= \frac{1}{10} \times \frac{22}{7} \times \left(\frac{35}{2}\right)^2$$

$$= \frac{385}{4} \text{mm}^2$$

Hence, The area of each sector of the brooch is $\frac{385}{4} \text{mm}^2$.

10. An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella. [Use $\pi = \frac{22}{7}$]



Ans:

Given that,

Radius of the umbrella $= r = 45 \text{cm}$

There are 8 ribs in an umbrella.

The angle between two consecutive ribs is subtending $\frac{360^\circ}{8} = 45^\circ$ at the center of the assumed flat circle.

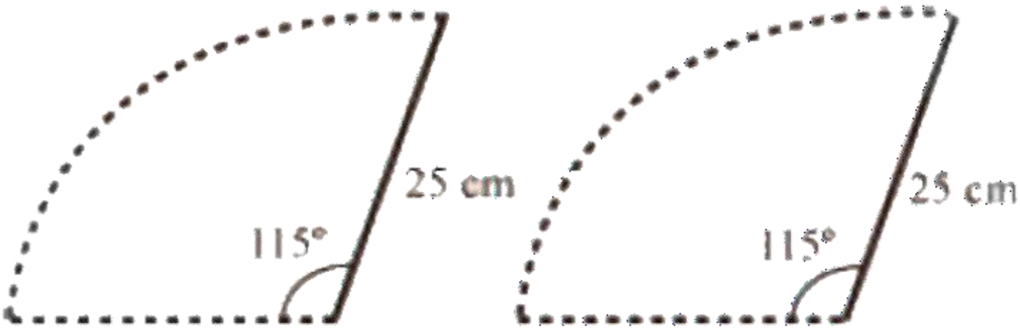
Area between two consecutive ribs of the assumed circle $= \frac{45^\circ}{360^\circ} \times \pi r^2$

$$= \frac{1}{8} \times \frac{22}{7} \times (45)^2$$

$$= \frac{22275}{28} \text{cm}^2$$

Hence, the area between the two consecutive ribs of the umbrella is $\frac{22275}{28} \text{cm}^2$.

11. A car has two wipers which do not overlap. Each wiper has blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades. [Use $\pi = \frac{22}{7}$]



Ans:

Given that,

Each blade of wiper will sweep an area of a sector of 115° in a circle of 25 cm radius.

Area of sector $= \frac{115^\circ}{360^\circ} \times \pi \times (25)^2$

$= \frac{158125}{252} \text{cm}^2$

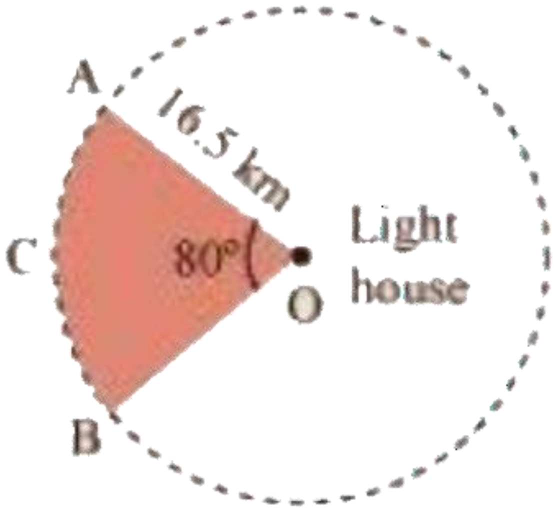
Area swept by 2 blades $= 2 \times \frac{158125}{252}$

$= \frac{158125}{126} \text{cm}^2$.

Therefore, the total area cleaned at each sweep of the blades is $\frac{158125}{126} \text{cm}^2$.

12. To warn ships for underwater rocks, a lighthouse spreads a red colored light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships warned. [Use $\pi = 3.14$]

Ans:

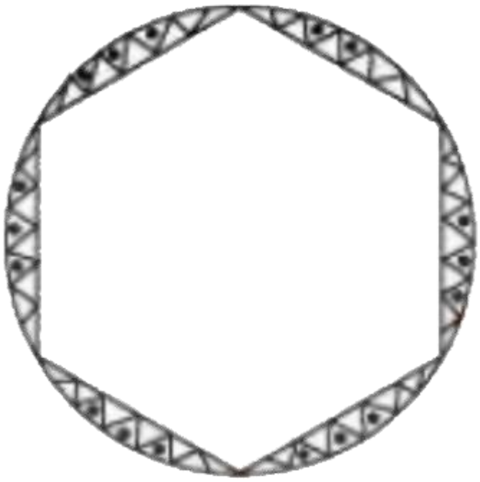


Given that,
The lighthouse spreads light across a sector (represented by shaded part in the figure) of 80° in a circle of 16.5 km radius.

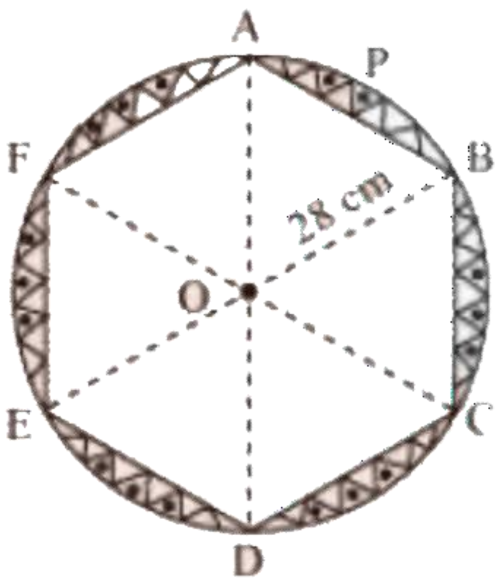
$$\text{Area of sector OACB} = \frac{80^\circ}{360^\circ} \times \pi r^2$$
$$= \frac{2}{9} \times 3.14 \times (16.5)^2$$
$$= 189.97 \text{km}^2$$

Hence, the area of the sea over which the ships are warned is 189.97km^2.

13. A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs.0.35 per $\mathbf{cm^2}$. [Use $\sqrt{3} = 1.7$]



Ans:



Given in the figure,
The designs are segments of the circle.
Radius of circle is 28cm.
Consider segment APB and chord AB is a side of the hexagon.
Each chord will substitute at $\frac{360^\circ}{6} = 60^\circ$ at the center of the circle.

In $\triangle OAB$,
Since, $OA = OB$

$\Rightarrow \angle OAB + \angle OBA + \angle AOB = 180^\circ$

$2\angle OAB = 180^\circ - 60^\circ = 120^\circ$

$\angle OAB = 60^\circ$

Therefore, $\triangle OAB$ is an equilateral triangle.

Area of $\triangle OAB = \frac{\sqrt{3}}{4} \times (\text{side})^2$

$= \frac{\sqrt{3}}{4} \times (28)^2$

$= 333.2 \text{cm}^2$

Area of sector OAPB $= \frac{60^\circ}{360^\circ} \times \pi r^2$

$= \frac{1}{6} \times \frac{22}{7} \times (28)^2$

$= \frac{1232}{3} \text{cm}^2$

Area of segment APBA = Area of sector OAPB − Area of ΔOAB

$= \left(\frac{1232}{3} - 333.2\right) \text{cm}^2$

Therefore, area of designs $= 6 \times \left(\frac{1232}{3} - 333.2\right) \text{cm}^2$

$= 464.8 \text{cm}^2$

Now, given that the Cost of making 1 cm^2 designs = Rs 0.35

Cost of making 464.76 cm^2 designs $= 464.8 \times 0.35 = 162.68$

Therefore, the cost of making such designs is Rs 162.68.

14. Tick the correct answer in the following: Area of a sector of angle p (in degrees) of a circle with radius R is

(A) $\frac{P}{180} \times 2\pi R$ (B) $\frac{P}{180} \times 2\pi R^2$ (C) $\frac{P}{180} \times \pi R$ (D) $\frac{P}{720} \times 2\pi R^2$

Ans:

We know that,

Area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi R^2$

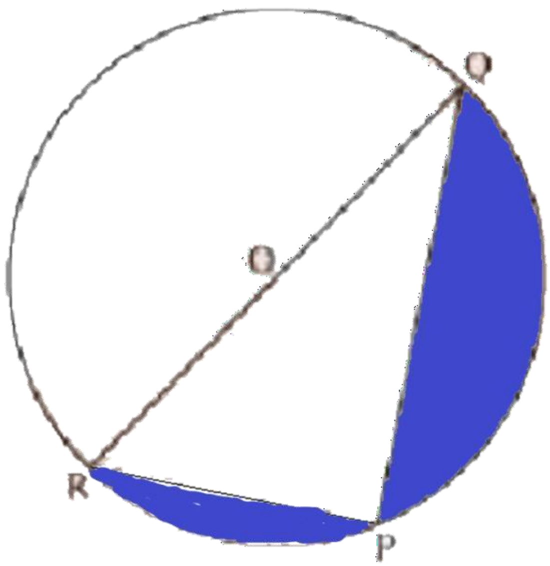
So, Area of sector of angle $P = \frac{P}{360^\circ}\left(\pi R^2\right)$

$= \left(\frac{P}{720^\circ}\right)\left(2\pi R^2\right)$

Hence, (D) is the correct answer.

Exercise 12.3

1. Find the area of the shaded region in the given figure, if PQ = 24 cm, PR = 7 cm and O is the center of the circle. [Use $\pi = \frac{22}{7}$]



Ans:

From the given figure,

RQ is the diameter of the circle which implies that $\angle RPQ = 90^\circ$

Thus,

By applying Pythagoras theorem in $\triangle PQR$,

$\Rightarrow RP^2 + PQ^2 = RQ^2$

$\Rightarrow (7)^2 + (24)^2 = RQ^2$

$\Rightarrow RQ = 25$

Thus, Radius of circle, $OR = \frac{RQ}{2} = \frac{25}{2}$

We know that, RQ is the diameter of the circle, it divides the circle in two equal parts.

So, for area of shaded region,

Area of semicircle $= \frac{1}{2}\pi r^2$

$$= \frac{1}{2}\pi\left(\frac{25}{2}\right)^2$$

$$= \frac{6875}{28}\text{cm}^2$$

Area of $\triangle PQR = \frac{1}{2} \times PQ \times PR$

$$= \frac{1}{2} \times 24 \times 7$$

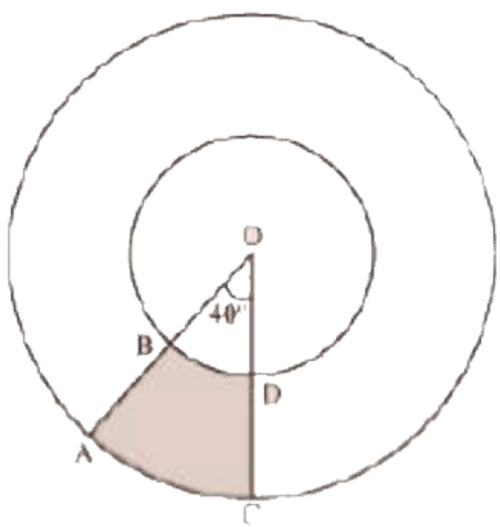
$$= 84\text{cm}^2$$

Area of shaded region = Area of semi - circle RPQOR − Area of $\triangle PQR$

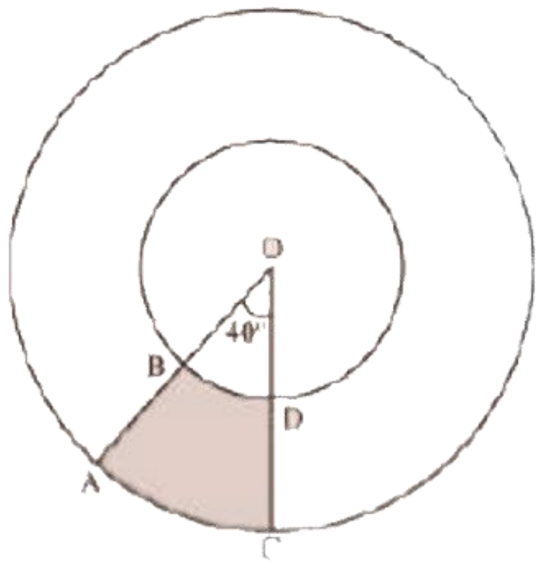
$$= \frac{6875}{28} - 84 = \frac{4532}{28}\text{cm}^2$$

Therefore, the area of the shaded region in the given figure is $\frac{4532}{28}\text{cm}^2$.

2. Find the area of the shaded region in the given figure, if radii of the two concentric circles with center O are 7 cm and 14 cm respectively and $\angle AOC = 40^\circ$. [Use $\pi = \frac{22}{7}$]



Ans:



Given that,
Radius of inner circle = 7 cm
Radius of outer circle = 14 cm
Angle subtended is 40°
Now,
Area of shaded region = Area of sector OAFC − Area of sector OBED

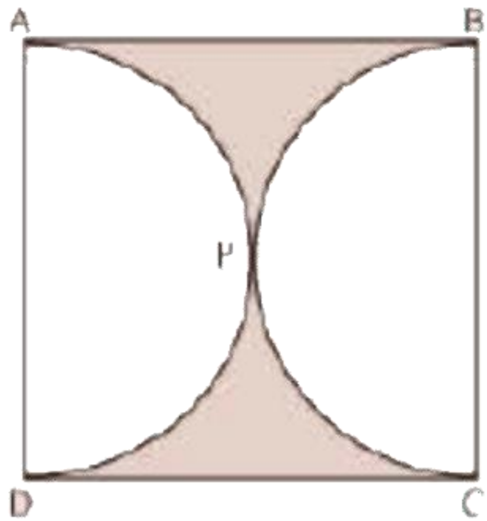
$$= \frac{40^\circ}{360^\circ} \times \pi(14)^2 - \frac{40^\circ}{360^\circ} \times \pi(7)^2$$

$$= \frac{616}{9} - \frac{159}{9}$$

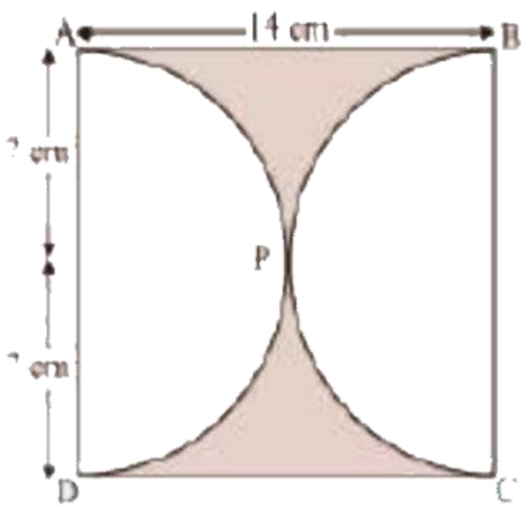
$$= \frac{154}{3}\text{cm}^2$$

Therefore, the area of the shaded region in the given figure is $\frac{154}{3}\text{cm}^2$.

3. Find the area of the shaded region in the given figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles. [Use $\pi = \frac{22}{7}$]



Ans:



From the above figure it is evident that the radius of each semi-circle is 7 cm.
For area of shaded region,

So, Area of each semi-circle $= \frac{1}{2}\pi r^2$

$$= \frac{1}{2} \times \frac{22}{7} \times (7)^2$$

$$= 77 cm^2$$

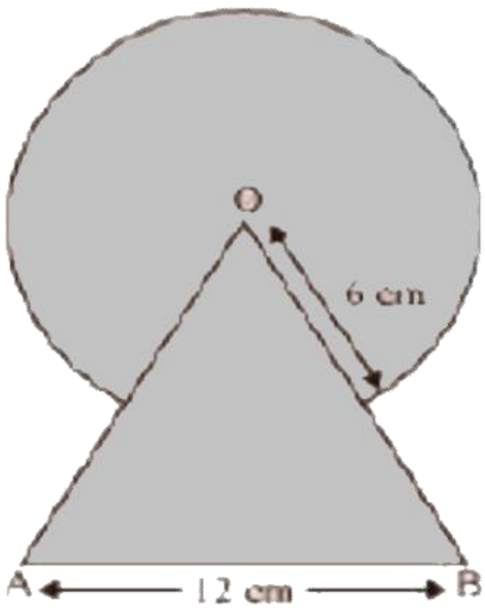
Area of square ABCD $= (side)^2 = (14)^2 = 196 cm^2$

Area of the shaded region = Area of square ABCD − Area of semi-circle APD − Area of semi-circle BPC

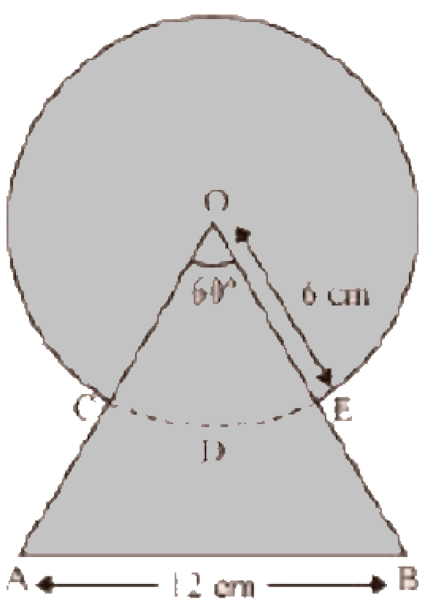
$= 196 - 77 - 77 = 42\text{cm}^2$

Therefore, the area of the shaded region in the given figure is 42cm^2.

4. Find the area of the shaded region in the given figure, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as center. [Use $\pi = \frac{22}{7}$]



Ans:



Given that,
Radius of the circle is 6cm
We know that each interior angle of an equilateral triangle is of measure 60°.
For area of shaded region,

Area of sector OCDE $= \frac{60^\circ}{360^\circ}\pi r^2$
$= \frac{1}{6} \times \frac{22}{7} \times (6)^2$
$= \frac{132}{7} cm^2$

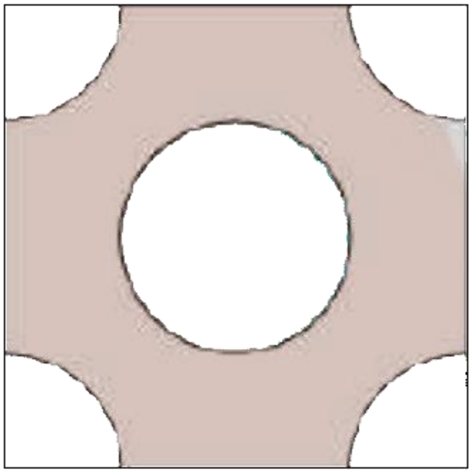
Area of $\triangle$OAB $= \frac{\sqrt{3}}{4}(12)^2$
$= 36\sqrt{3} cm^2$

Area of circle $= \pi r^2$
$= \frac{22}{7} \times (6)^2$
$= \frac{792}{7} cm^2$

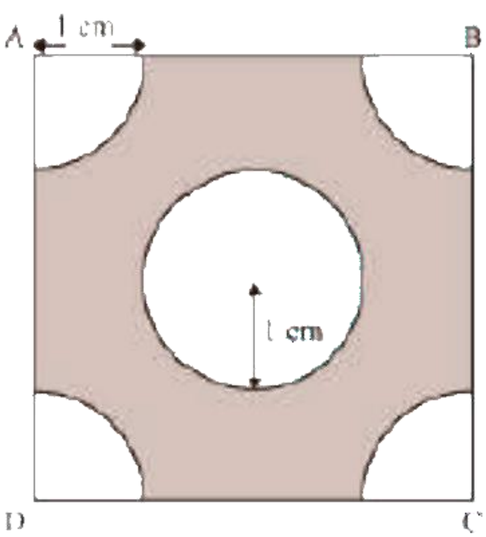
Area of shaded region = Area of ΔOAB + Area of circle − Area of sector OCDE
$= 36\sqrt{3} + \frac{792}{7} - \frac{132}{7}$
$= \left(36\sqrt{3} + \frac{660}{7}\right) cm^2$

Hence, the area of the shaded region in the given figure $\left(36\sqrt{3} + \frac{660}{7}\right) cm^2$.

5. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in the given figure. Find the area of the remaining portion of the square. [Use $\pi = \frac{22}{7}$]



Ans:



It is evident from the above figure that each quadrant is a sector of 90° in a circle of 1 cm radius.

For area of shaded region,

$$\text{Area of each quadrant } = \frac{90^\circ}{360^\circ}\pi r^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times (1)^2$$

$= \frac{22}{28} cm^2$

Area of square $= (\text{side})^2$

$= (4)^2$

$= 16 cm^2$

Area of circle $= \pi r^2$

$= \pi(1)^2 = \frac{22}{7} cm^2$

Area of the shaded region = Area of square – Area of circle – (4 × Area of quadrant)

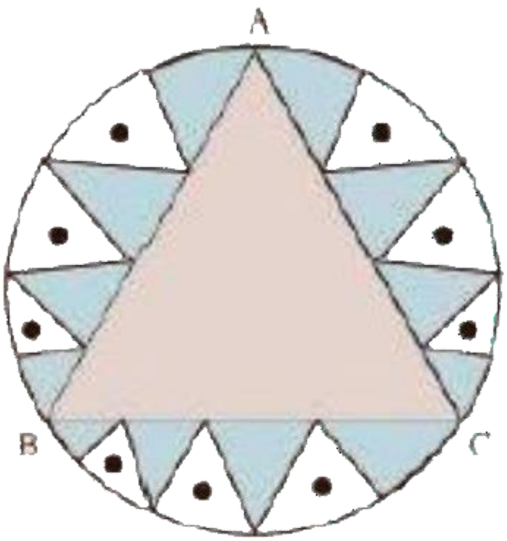
$= 16 - \frac{22}{7} - \left(4 \times \frac{22}{28}\right)$

$= 16 - \frac{44}{7}$

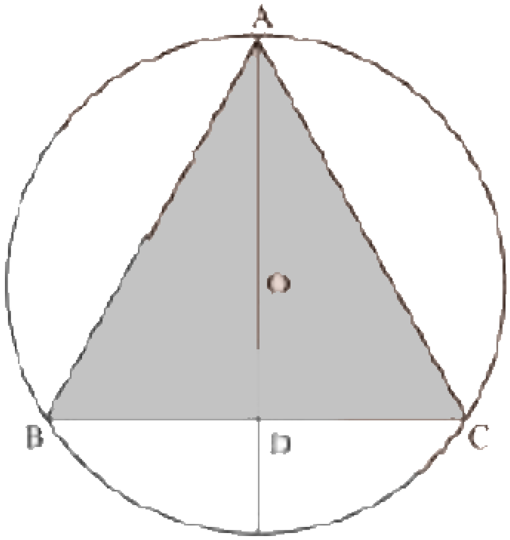
$= \frac{68}{7} cm^2$

Therefore, the area of the remaining portion of the square is $\frac{68}{7} cm^2$.

6. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in the given figure. Find the area of the design (Shaded region). [Use $\pi = \frac{22}{7}$]



Ans:



Given that,

Radius of circle = r = 32 cm

AD is the median of $\triangle ABC$

$$OA = \frac{2}{3}AD$$

$$AD = 48cm$$

In $\triangle ABD$,

Using Pythagoras Theorem,

$$AB^2 = AD^2 + BD^2$$

$$AB^2 = (48)^2 + \left(\frac{BC}{2}\right)^2$$

$$AB^2 = (48)^2 + \left(\frac{AB}{2}\right)^2$$

$$\Rightarrow \frac{3AB^2}{4} = (48)^2$$

$$\Rightarrow AB = \frac{48 \times 2}{\sqrt{3}} = \frac{96}{\sqrt{3}}$$

$$= 32\sqrt{3} \text{ cm}$$

For area of design,

Area of equilateral triangle $= \frac{\sqrt{3}}{4} \times (32)^2 \times 3$

$= 96 \times 8 \times \sqrt{3}$

$= 768\sqrt{3} cm^2$

Area of circle $= \pi r^2$

$= \frac{22}{7} \times (32)^2$

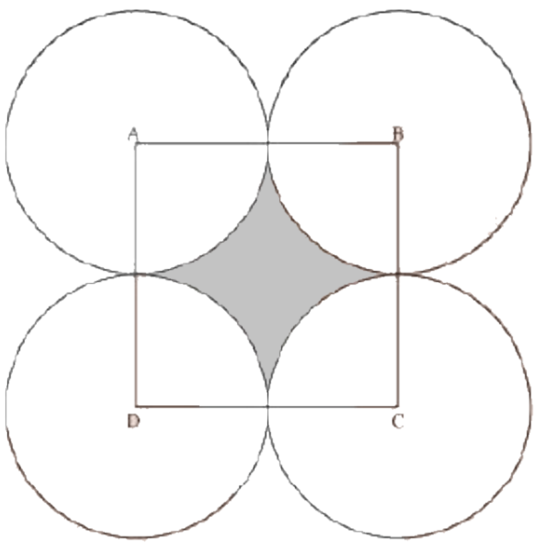
$= \frac{22528}{7} cm^2$

Area of design = Area of circle − Area of $\triangle ABC$

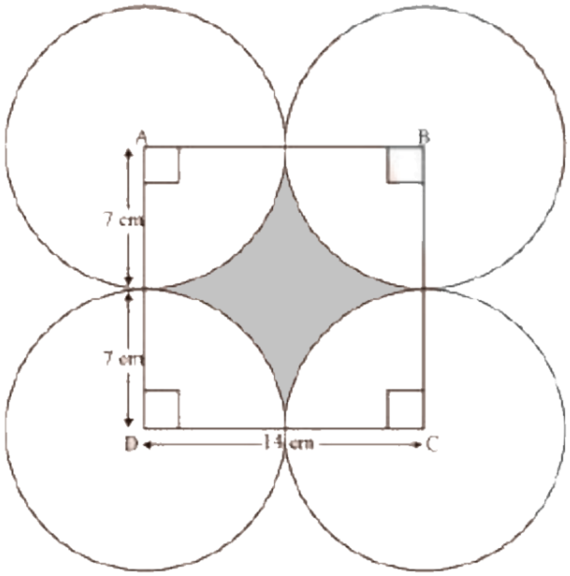
$= \left(\frac{22528}{7} - 768\sqrt{3}\right) cm^2$

Hence, the area of the design (Shaded region) is $\left(\frac{22528}{7} - 768\sqrt{3}\right) cm^2$.

7. In the given figure, ABCD is a square of side 14 cm. With centers A, B, C and D, four circles are drawn such that each circle touches externally two of the remaining three circles. Find the area of the shaded region. [Use $\pi = \frac{22}{7}$]



Ans:



It is evident that,
Area of each of the 4 sectors is equal to each other
Sector of 90° in a circle of 7 cm radius.
For the area of shaded region,

Area of each sector $= \frac{90^\circ}{360^\circ}\pi(7)^2$

$= \frac{1}{4} \times \frac{22}{7} \times (7)^2$

$= \frac{77}{2}\text{cm}^2$

Area of square ABCD $= (\text{side})^2$

$= (14)^2 = 196\text{cm}^2$

Area of shaded portion = Area of square ABCD – (4 × Area of each sector)

$= 196 - \left(4 \times \frac{77}{2}\right)$

$= 42\text{cm}^2$

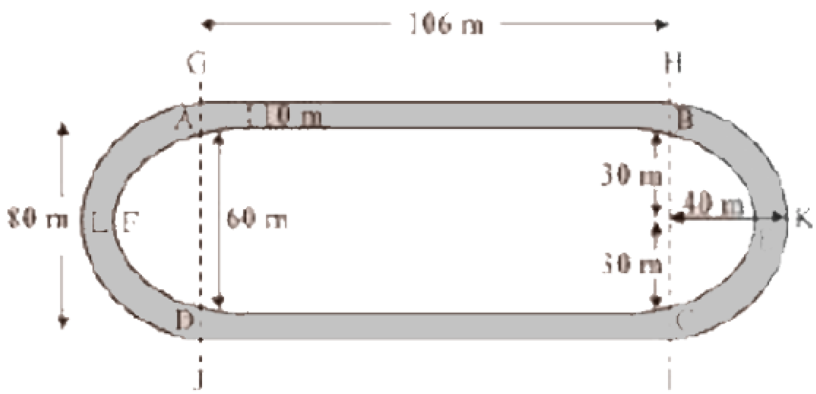
Therefore, the area of shaded portion is 42cm^2.

8. The given figure depicts a racing track whose left and right ends are semicircular.

The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, [Use $\pi = \frac{22}{7}$]

find

Ans:



(i) The distance around the track along its inner edge

Ans: For inner edge,

Radius = r = 30cm

Distance around the track along its inner edge = AB + arc BEC + CD + arc DFA

$$= 106 + \left(\frac{1}{2} \times 2\pi r\right) + 106 + \left(\frac{1}{2} \times 2\pi r\right)$$

$$= 212 + \left(\frac{1}{2} \times 2 \times \frac{22}{7} \times 30\right) + \left(\frac{1}{2} \times 2 \times \frac{22}{7} \times 30\right)$$

$$= 212 + \left(2 \times \frac{22}{7} \times 30\right)$$

$$= \frac{2804}{7} \text{ m}$$

Hence, distance around the track along its inner edge is $\frac{2804}{7}$ m.

(ii) The area of the track

Ans: Radius of inner edge = 30cm

Radius of outer edge = 40cm

Area of the track = (Area of GHIJ − Area of ABCD) + (Area of semi-circle HKI – Area of semi- circle BEC) + (Area of semi-circle GLJ − Area of semicircle AFD)

$$= (106\times 80) - (106\times 60) + \left(\frac{1}{2}\times\frac{22}{7}\times(40)^2\right) - \left(\frac{1}{2}\times\frac{22}{7}\times(30)^2\right) + \left(\frac{1}{2}\times\frac{22}{7}\times(40)^2\right) - \left(\frac{1}{2}\times\frac{22}{7}\times(30)^2\right)$$

$$= 106(80-60) + \left(\frac{22}{7}\times(40)^2\right) - \left(\frac{22}{7}\times(30)^2\right)$$

$$= 2120 + \frac{22}{7}\left[(40)^2 - (30)^2\right]$$

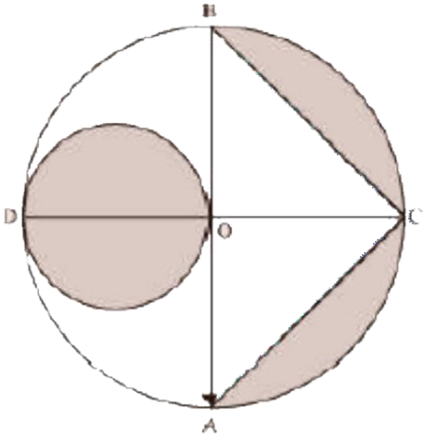
$$= 2120 + 2200$$

$$= 4320\text{m}^2$$

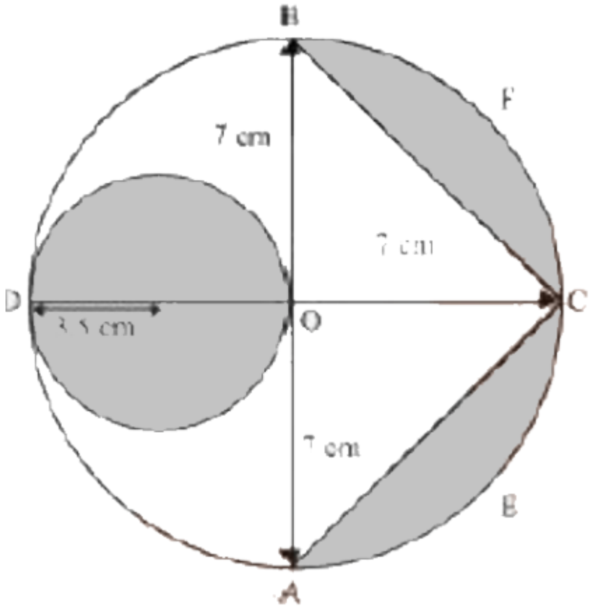
Therefore, the area of the track is 4320m^2.

9. In the given figure, AB and CD are two diameters of a circle (with center O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.

[Use $\pi = \frac{22}{7}$]



Ans:



Given that,

Radius of larger circle $= r_1 = 7$ cm

Radius of smaller circle $= r_2 = \frac{7}{2}$ cm

For area of shaded region,

Area of smaller circle $= \pi r_2^2$

$$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

$$= \frac{77}{2} \text{ cm}^2$$

Area of semi-circle AECFB of larger circle $= \frac{1}{2} \pi r_2^2$

$$= \frac{1}{2} \times \frac{22}{7} \times (7)^2$$

$$= \frac{77}{2} \text{ cm}^2$$

Area of $\triangle ABC = \frac{1}{2} \times AB \times OC$

$$= \frac{1}{2} \times 14 \times 7$$

$= 49 cm^2$

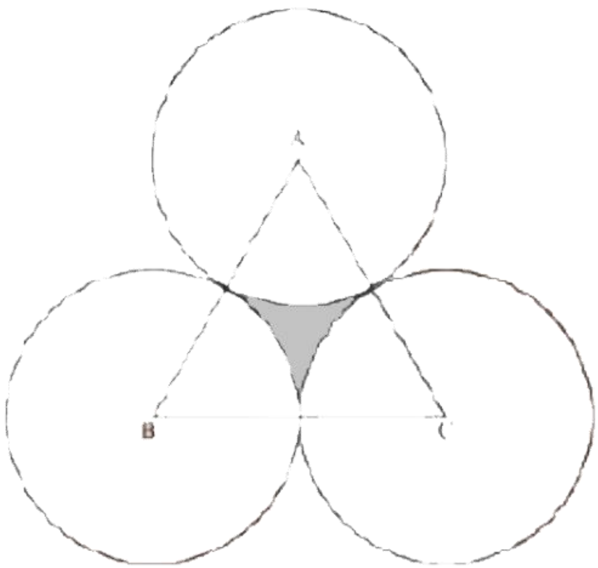
Area of the shaded region = Area of smaller circle + Area of semi-circle AECFB − Area of $\triangle ABC$

$$= \frac{77}{2} + 77 - 49$$

$$= 28 + 38.5 = 66.5 cm^2$$

Therefore, the area of shaded region is $66.5 cm^2$.

10. The area of an equilateral triangle ABC is $17320.5 cm^2$. With each vertex of the triangle as center, a circle is drawn with radius equal to half the length of the side of the triangle (See the given figure). Find the area of shaded region. [Use $\pi = 3.14$ and $\sqrt{3} = 1.73205$]



Ans:

Let the side of the equilateral triangle be a.

Given that,

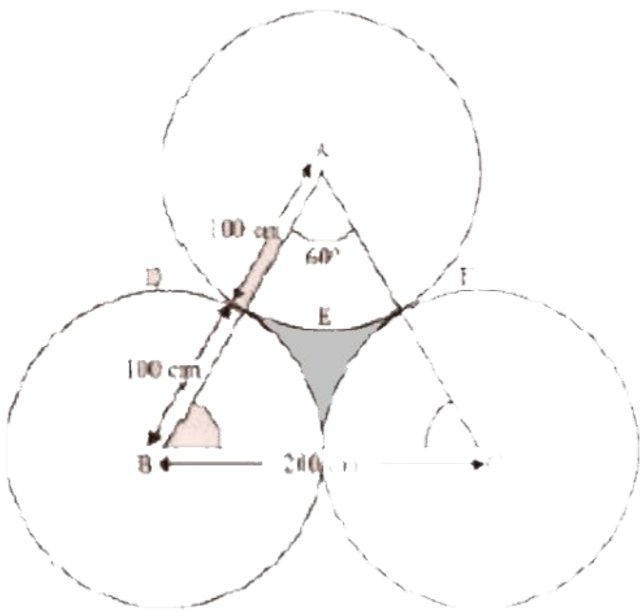
Area of equilateral triangle $= 17320.5 cm^2$

$$\Rightarrow \frac{\sqrt{3}}{4}(a)^2 = 17320.5$$

$$\Rightarrow \frac{1.73205}{4}a^2 = 17320.5$$

$$\Rightarrow a^2 = 4 \times 10000$$

$$\Rightarrow a = 200 cm$$



It is evident from the figure that, each sector is of measure 60°

Area of sector ADEF $= \frac{60^\circ}{360^\circ} \times \pi \times r^2$

$= \frac{1}{6} \times \pi \times (100)^2$

$= \frac{3.14 \times 10000}{6}$

$= \frac{15700}{3} \text{cm}^2$

Area of shaded region = Area of equilateral triangle – (3 × Area of each sector)

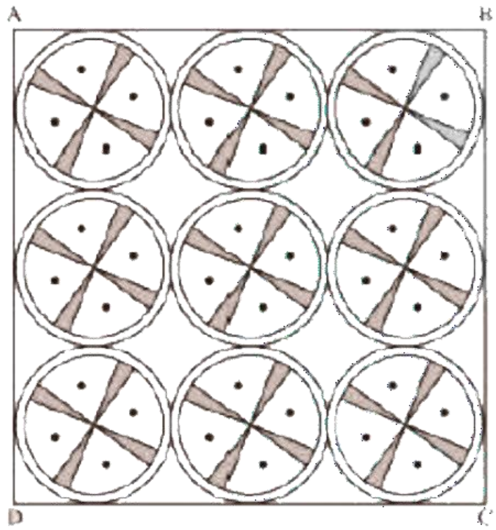
$= 17320.5 - 3 \times \frac{15700}{3}$

$= 17320.5 - 15700$

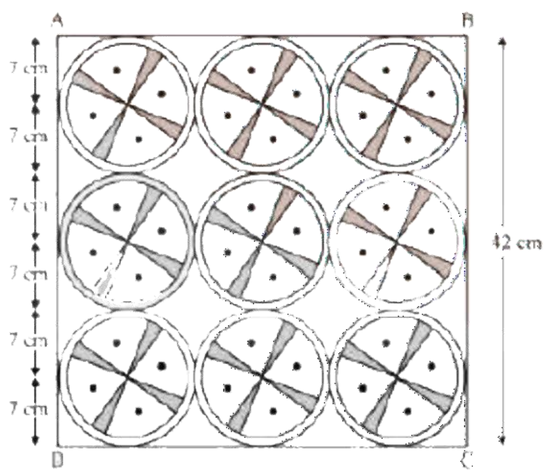
$= 1620.5 \text{cm}^2$

Therefore, area of given shaded region is 1620.5cm^2.

11. On a square handkerchief, nine circular designs each of radius 7 cm are made (see the given figure). Find the area of the remaining portion of the handkerchief. [Use $\pi = \frac{22}{7}$]



Ans:



It is evident from the above figure, that the side of the square is 42 cm.
So, for area of the remaining portion of handkerchief,

Area of square $= (\text{side})^2 = (42)^2$

$= 1764\text{cm}^2$

Area of each circle $= \pi r^2 = \frac{22}{7} \times (7)^2$

$= 154\text{cm}^2$

Area of 9 circles $= 9 \times 154$

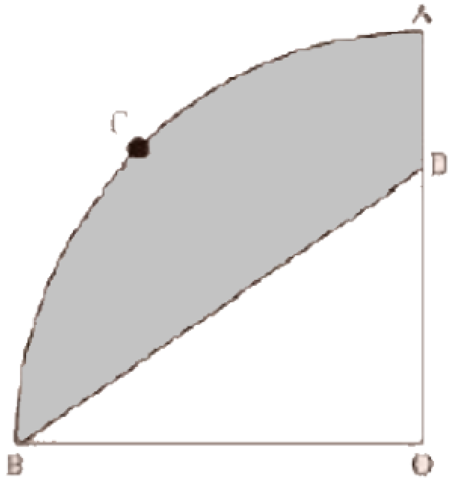
$= 1386\text{cm}^2$

Area of the remaining portion of the handkerchief $= 1764 - 1386$

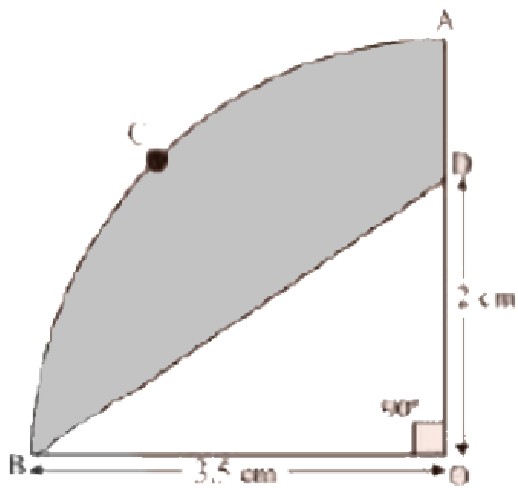
$= 378\text{cm}^2$

Therefore, the area of the remaining portion of the handkerchief is $378 cm^2$.

12. In the given figure, OACB is a quadrant of circle with center O and radius 3.5 cm. If OD = 2 cm, find the area of the [Use $\pi = \frac{22}{7}$]



Ans:



Given that radius is 3.5cm.

(i) Quadrant OACB

Ans:

Since OACB is a quadrant, the angle at O is 90°.

Area of quadrant OACB $= \frac{90^\circ}{360^\circ} \times \pi r^2$

$$=\frac{1}{4}\times\frac{22}{7}\times(3.5)^2$$
$$=\frac{1}{4}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2$$
$$=\frac{77}{8}\text{cm}^2$$

Therefore, the area of quadrant OACB is $\frac{77}{8}\text{cm}^2$.

(ii) Shaded region

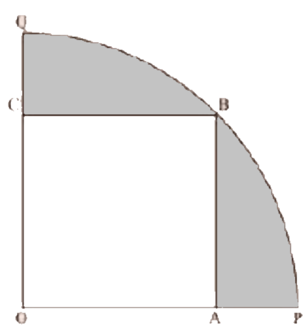
Ans: For the area of the shaded region,

Area of $\triangle$OBD $=\frac{1}{2}\times \text{OB}\times \text{OD}$
$$=\frac{1}{2}\times 3.5\times 2$$
$$=\frac{7}{2}\text{cm}^2$$

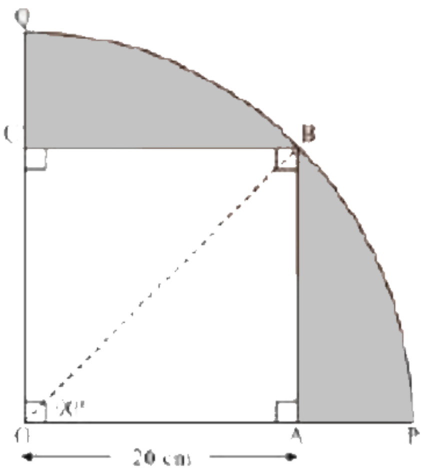
Area of the shaded region = Area of quadrant OACB − Area of $\triangle$OBD
$$=\frac{77}{8}-\frac{7}{2}$$
$$=\frac{49}{8}\text{cm}^2.$$

Hence, the area of the shaded region in the given figure is $\frac{49}{8}\text{cm}^2$.

13. In the given figure, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. [Use $\pi = 3.14$]



Ans:



For radius

In $\triangle OAB$,

Using Pythagoras Theorem,

$$OB^2 = OA^2 + OB^2$$
$$= (20)^2 + (20)^2$$
$$\Rightarrow OB = 20\sqrt{2}$$

Radius of circle $= r = 20\sqrt{2}$ cm

For the area of shaded region,

Area of quadrant OPBQ $= \frac{90^\circ}{360^\circ} \times 3.14 \times \left(20\sqrt{2}\right)^2$

$$= \frac{1}{4} \times 3.14 \times 800$$
$$= 628\text{cm}^2$$

Area of square OABC $= (\text{side})^2$

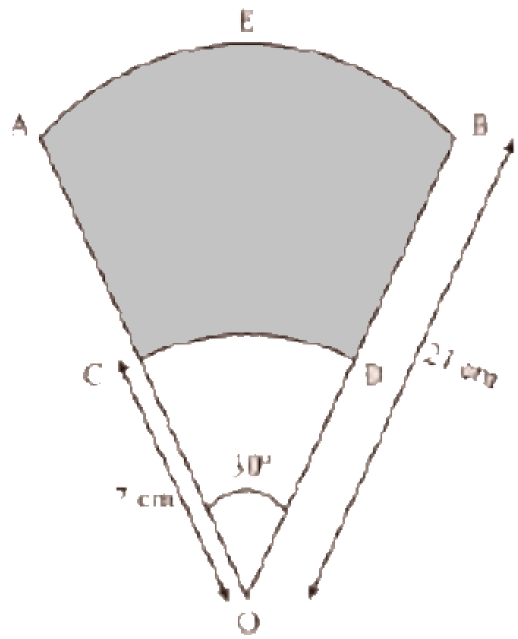
$$= (20)^2$$
$$= 400\text{cm}^2$$

Area of shaded region = Area of quadrant OPBQ − Area of square OABC

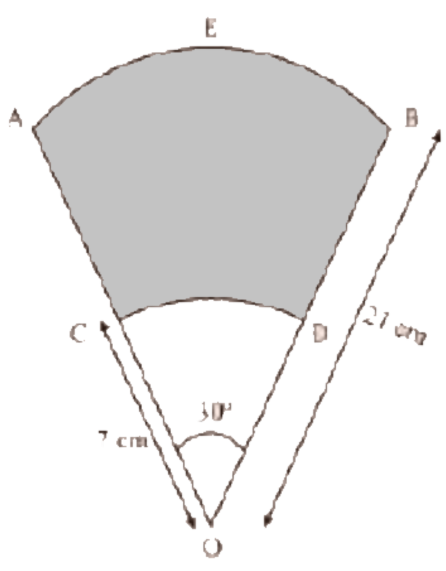
$$= (628 - 400)\text{cm}^2$$
$$= 228\text{cm}^2$$

Therefore, the area of the shaded region is 228cm^2.

14. AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and center O (see the given figure). If $\angle AOB = 30^\circ$, find the area of the shaded region. [Use $\pi = \frac{22}{7}$]



Ans:



Given that,
Radius for sector OAEB = 21cm
Radius for sector OCFD = 7cm

Angle subtended is 30°

Area of the shaded region = Area of sector OAEB − Area of sector OCFD

$$=\left[\frac{30^\circ}{360^\circ}\times\pi\times(21)^2\right]-\left[\frac{30^\circ}{360^\circ}\times\pi\times(7)^2\right]$$

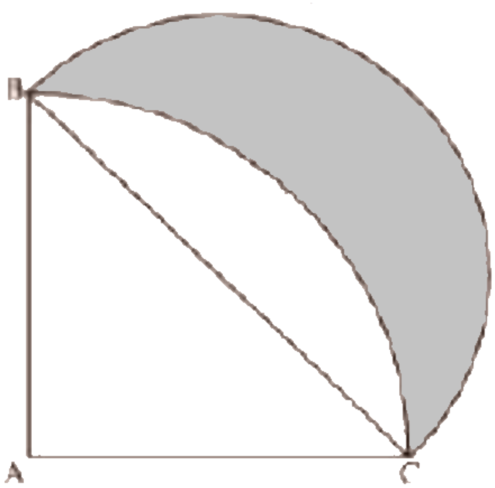
$$=\frac{1}{12}\pi\left[(21-7)(21+7)\right]$$

$$=\frac{22\times14\times28}{12\times7}$$

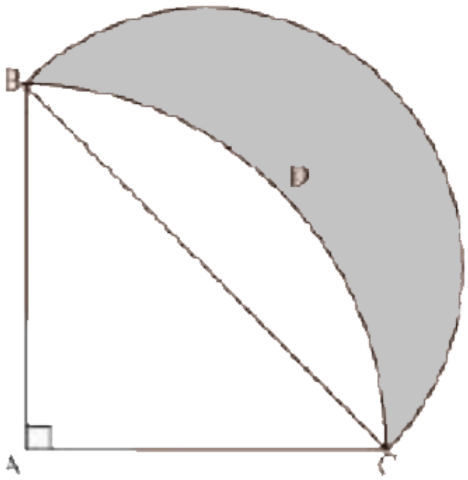
$$=\frac{308}{3}\text{cm}^2$$

Therefore, the area of shaded region is $\frac{308}{3}\text{cm}^2$.

15. In the given figure, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region. [Use $\pi=\frac{22}{7}$]



Ans:



Given that,

Radius of circle = r = 14cm

As ABC is a quadrant of the circle, $\angle BAC$ will be of measure 90°.

In $\triangle ABC$,

Using Pythagoras Theorem,

$\Rightarrow BC^2 = AC^2 + AB^2$

$= (14)^2 + (14)^2$

$\Rightarrow BC = 14\sqrt{2}$

Radius of semi-circle drawn on BC = $r_1 = = \frac{14\sqrt{2}}{2} = 7\sqrt{2}cm$

Now, for area of the shaded region,

Area of $\triangle ABC = \frac{1}{2} \times AB \times AC$

$= \frac{1}{2} \times (14)^2$

$= 98cm^2$

Area of sector ABDC $= \frac{90^\circ}{360^\circ} \times \pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times (14)^2$

$= 154cm^2$

Area of semi-circle drawn on BC $= \frac{1}{2} \times \pi \times r_1^2$

$$= \frac{1}{2} \times \frac{22}{7} \times (7\sqrt{2})^2$$

$$= 154 cm^2$$

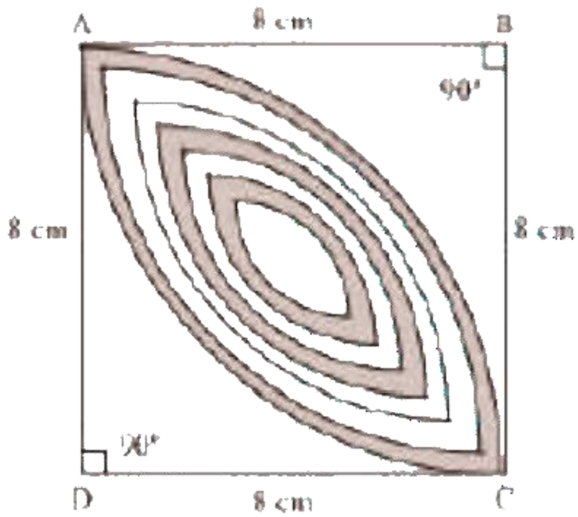
Area of shaded region = Area of semi-circle on BC – (Area of sector ABDC – Area of ΔABC)

$$= 154 - (154 - 98)$$

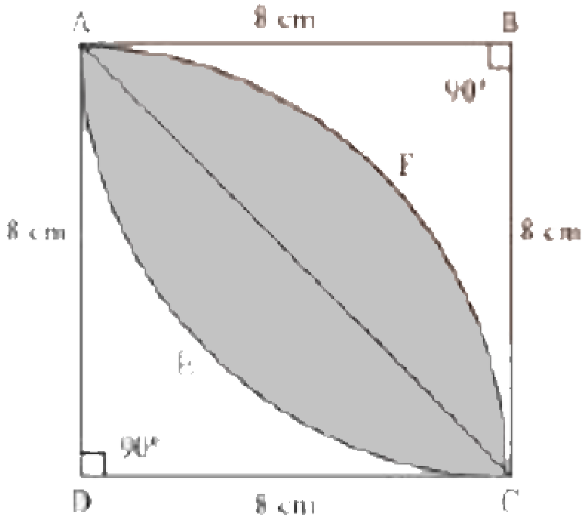
$$= 98 cm^2$$

Hence, the area of the shaded region of the given figure is $98 cm^2$.

16. Calculate the area of the designed region in the given figure common between the two quadrants of circles of radius 8 cm each. [Use $\pi = \frac{22}{7}$]



Ans:



Given that ,
Radius of each circle is 8cm
The designed area is the common region between two sectors BAEC and DAFC.
For the area of the designed region,

Area of sector $= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (8)^2$

$= \frac{1}{4} \times \frac{22}{7} \times 64$

$= \frac{22 \times 16}{7}$

$= \frac{352}{7} \text{cm}^2$

Area of $\triangle BAC = \frac{1}{2} \times BA \times BC$

$= \frac{1}{2} \times (8)^2$

$= 32 \text{cm}^2$

Area of the designed portion = 2 × (Area of segment AEC) = 2 × (Area of sector BAEC − Area of $\triangle$BAC)

$= 2 \times \left(\frac{352}{7} - 32 \right)$

$$= \frac{2 \times 128}{7}$$

$$= \frac{256}{7} \text{cm}^2$$

Hence, the area of the designed region is $\frac{256}{7} \text{cm}^2$.