NCERT Solutions for Class 10

Maths

Chapter 4 – Quadratic Equations

Exercise 4.1

1. Check whether the following are quadratic equations:

i. $\mathbf{(x+1)^2=2(x-3)}$

Ans: $(x+1)^2=2(x-3)$

$\Rightarrow x^2+2x+1=2x-6$

$\Rightarrow x^2+7=0$

Since, it is in the form of $ax^2+bx+c=0$.
Therefore, the given equation is a quadratic equation.

ii. $\mathbf{x^2-2x=(-2)(3-x)}$

Ans: $x^2 \quad -2x=(-2)(3-x)$

$\Rightarrow x^2-2x=-6+2x$

$\Rightarrow x^2-4x+6=0$

Since, it is in the form of $ax^2+bx+c=0$.
Therefore, the given equation is a quadratic equation.

iii. $\mathbf{(x-2)(x+1)=(x-1)(x+3)}$

Ans: $(x-2)(x+1)=(x-1)(x+3)$

$\Rightarrow x^2-x-2=x^2+2x-3$

$\Rightarrow 3x-1=0$

Since, it is not in the form of $ax^2+bx+c=0$.
Therefore, the given equation is not a quadratic equation.

iv. $\mathbf{(x-3)(2x+1)=x(x+5)}$

Ans: $(x-3)(2x+1)=x(x+5)$

$\Rightarrow 2x^2-5x-3=x^2+5x$

$\Rightarrow x^2-10x-3=0$

Since, it is in the form of $ax^2+bx+c=0$.
Therefore, the given equation is a quadratic equation.

v. $(2x-1)(x-3)=(x+5)(x-1)$

Ans: $(2x-1)(x-3)=(x+5)(x-1)$

$\Rightarrow 2x^2-7x+3=x^2+4x-5$

$\Rightarrow x^2-11x+8=0$

Since, it is in the form of $ax^2+bx+c=0$.

Therefore, the given equation is a quadratic equation.

vi. $x^2+3x+1=(x-2)^2$

Ans: $x^2+3x+1=(x-2)^2$

$\Rightarrow x^2+3x+1=x^2+4-2x$

$\Rightarrow 7x-3=0$

Since, it is not in the form of $ax^2+bx+c=0$.

Therefore, the given equation is not a quadratic equation.

vii. $(x+2)^3=2x(x^2-1)$

Ans: $(x+2)^3=2x(x^2-1)$

$\Rightarrow x^3+8+6x^2+12x=2x^3-2x$

$\Rightarrow x^3-14x-6x^2-8=0$

Since, it is not in the form of $ax^2+bx+c=0$.

Therefore, the given equation is not a quadratic equation.

viii. $x^3-4x^2-x+1=(x-2)^3$

Ans: $x^3-4x^2-x+1=(x-2)^3$

$\Rightarrow x^3-4x^2-x+1=x^3-8-6x^2+12x$

$\Rightarrow 2x^2-13x+9=0$

Since, it is in the form of $ax^2+bx+c=0$.

Therefore, the given equation is a quadratic equation.

2. Represent the following situations in the form of quadratic equations.

i. The area of a rectangular plot is $528\ m^2$. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

Ans: Let the breath of the plot be x m.

Thus, length would be-

$\text{Length}=(2x+1)\text{m}$

Hence, Area of rectangle $=\text{Length}\times\text{breadth}$

So, $528=x(2x+1)$

$\Rightarrow 2x^2-x-528=0$

ii. The product of two consecutive positive integers is 306. We need to find the integers.

Ans: Let the consecutive integers be x and $x+1$.

Thus, according to question-

$x(x+1)=306$

$\Rightarrow x^2+x-306=0$

iii. Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

Ans: Let Rohan's age be x.

Hence, his mother's age is $x+26$.

Now, after 3 years.

Rohan's age will be $x+3$.

His mother's age will be $x+29$.

So, according to question-

$(x+3)(x+29)=360$

$\Rightarrow x^2+3x+29x+87=360$

$\Rightarrow x^2+32x-273=0$

iv. A train travels a distance of 480 kmat a uniform speed. If the speed had been 8km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Ans: Let the speed of train be x km/h.

Thus, time taken to travel 482 km is $\frac{480}{x}$ hrs.

Now, let the speed of train $=(x-8)$km/h.

Therefore, time taken to travel 482 km is $\left(\frac{480}{x}+3\right)$hrs.

Hence, speed×time=distance

i.e $(x-8)\left(\frac{480}{x}+3\right)=480$

$\Rightarrow 480+3x-\frac{3840}{x}-24=480$

$\Rightarrow 3x-\frac{3840}{x}=24$

$\Rightarrow 3x^2-24x-3840=0$

$\Rightarrow x^2-8x-1280=0$

Exercise 4.2

1. Find the roots of the following quadratic equations by factorisation:

i. $x^2-3x-10=0$

Ans: $x^2-3x-10=0$

$\Rightarrow x^2-5x+2x-10$

$\Rightarrow x(x-5)+2(x-5)$

$\Rightarrow (x-5)(x+2)$

Therefore, roots of this equation are –

$x-5=0$ or $x+2=0$

i.e $x=5$ or $x=-2$

ii. $2x^2+x-6=0$

Ans: $2x^2+x-6=0$

$\Rightarrow 2x^2+4x-3x-6$

$\Rightarrow 2x(x+2)-3(x+2)$

$\Rightarrow (x+2)(2x-3)$

Therefore, roots of this equation are –

$x+2=0$ or $2x-3=0$

i.e $x=-2$ or $x=\frac{3}{2}$

iii. $\sqrt{2}x^2+7x+5\sqrt{2}=0$

Ans: $\sqrt{2}x^2+7x+5\sqrt{2}=0$

$\Rightarrow \sqrt{2}x^2+5x+2x+5\sqrt{2}$

$\Rightarrow x(\sqrt{2}x+5)+\sqrt{2}(\sqrt{2}x+5)$

$\Rightarrow (\sqrt{2}x+5)(x+\sqrt{2})$

Therefore, roots of this equation are –

$\sqrt{2}x+5=0$ or $x+\sqrt{2}=0$

i.e $x=\frac{1}{10}$ or $x=\frac{1}{10}$

2. Question

i. John and Jivanti together have 45marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. Find out how many marbles they had to start with.

Ans: Let the number of john's marbles be x.

Thus, number of Jivanti's marble be $45-x$.

According to question i.e,

After losing 5 marbles.

Number of john's marbles be $x-5$

And number of Jivanti's marble be $40-x$.

Therefore, $\left(x-5\right)\left(40-x\right)=124$

$\Rightarrow x^2-45x+324=0$

$\Rightarrow x^2-36x-9x+324=0$

$\Rightarrow x\left(x-36\right)-9\left(x-36\right)=0$

$\Rightarrow \left(x-36\right)\left(x-9\right)=0$

So now,

Case 1- If $x-36=0$ i.e $x=36$

So, the number of john's marbles be 36.

Thus, number of Jivanti's marble be 9.

Case 2- If $x-9=0$ i.e $x=9$

So, the number of john's marbles be 9.

Thus, number of Jivanti's marble be 36.

ii. A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55minus the number of toys produced in a day. On a particular day, the total cost of production was Rs 750. Find out the number of toys produced on that day.

Ans: Let the number of toys produced be x.

Therefore, Cost of production of each toy be $Rs\left(55-x\right)$.

Thus, $\left(55-x\right)x=750$

$\Rightarrow x^2-55x+750=0$

$\Rightarrow x^2-25x-30x+750=0$

$\Rightarrow x\left(x-25\right)-30\left(x-25\right)=0$

$\Rightarrow \left(x-25\right)\left(x-30\right)=0$

Case 1- If $x-25=0$ i.e $x=25$

So, the number of toys be 25.

Case 2- If $x-30=0$ i.e $x=30$
So, the number of toys be 30.

3. Find two numbers whose sum is 27 and product is 182.

Ans: Let the first number be x,
Thus, the second number be $27-x$.
Therefore,

$x(27-x)=182$

$\Rightarrow x^2-27x+182=0$

$\Rightarrow x^2-13x-14x+182=0$

$\Rightarrow x(x-13)-14(x-13)=0$

$\Rightarrow (x-13)(x-14)=0$

Case 1- If $x-13=0$ i.e $x=13$
So, the first number be 13,
Thus, the second number be 14.
Case 2- If $x-14=0$ i.e $x=14$
So, the first number be 14.
Thus, the second number be 13.

4. Find two consecutive positive integers, sum of whose squares is 365.

Ans: Let the consecutive positive integers be x and $x+1$.

Thus, $x^2+(x+1)^2=365$

$\Rightarrow x^2+x^2+1+2x=365$

$\Rightarrow 2x^2+2x-364=0$

$\Rightarrow x^2+x-182=0$

$\Rightarrow x^2+14x-13x-182=0$

$\Rightarrow x(x+14)-13(x+14)=0$

$\Rightarrow (x+14)(x-13)=0$

Case 1- If $x+14=0$ i.e $x=-14$.
This case is rejected because number is positive.
Case 2- If $x-13=0$ i.e $x=13$
So, the first number be 13.
Thus, the second number be 14.
Hence, the two consecutive positive integers are 13 and 14.

5. The altitude of a right triangle is 7 cmless than its base. If the hypotenuse is 13 cm, find the other two sides.
Ans: Let the base of the right-angled triangle be x cm .
Its altitude be $(x-7)$cm .
Thus, by pythagores theorem-
$\text{base}^2 + \text{altitude}^2 = \text{hypotenuse}^2$

$\therefore x^2 + (x-7)^2 = 13^2$

$\Rightarrow x^2 + x^2 + 49 - 14x = 169$

$\Rightarrow 2x^2 - 14x - 120 = 0$

$\Rightarrow x^2 - 7x - 60 = 0$

$\Rightarrow x^2 + 12x + 5x - 60 = 0$

$\Rightarrow x(x-12) + 5(x-12) = 0$

$\Rightarrow (x-12)(x+5) = 0$

Case 1- If $x-12=0$ i.e $x=12$.
So, the base of the right-angled triangle be 12 cm and Its altitude be 5cm
Case 2- If $x+5=0$ i.e $x=-5$
This case is rejected because side is always positive.
Hence, the base of the right-angled triangle be 12 cm and Its altitude be 5cm.

6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article.
Ans: Let the number of articles produced be x.
Therefore, cost of production of each article be $Rs(2x+3)$.

Thus, $x(2x+3) = 90$

$\Rightarrow 2x^2 + 3x - 90 = 0$

$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$

$\Rightarrow x(2x+15) - 6(2x+15) = 0$

$\Rightarrow (2x+15)(x-6) = 0$

Case 1- If $2x-15=0$ i.e $x = \frac{-15}{2}$.
This case is rejected because number of articles is always positive.
Case 2- If $x-6=0$ i.e $x=6$

Hence, the number of articles produced be 6 .
Therefore, cost of production of each article be Rs15.

Exercise 4.3

1. Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

i. $2x^2-7x+3=0$

Ans: $2x^2-7x+3=0$

On dividing both sides of the equation by 2.

$\Rightarrow x^2-\frac{7}{2}x=-\frac{3}{2}$

$\Rightarrow x^2-2\left(\frac{7}{4}\right)x=-\frac{3}{2}$

On adding $\left(\frac{7}{4}\right)^2$ both sides of equation.

$\Rightarrow x^2-2\left(\frac{7}{4}\right)x+\left(\frac{7}{4}\right)^2=-\frac{3}{2}+\left(\frac{7}{4}\right)^2$

$\Rightarrow \left(x-\frac{7}{4}\right)^2=\frac{49}{16}-\frac{3}{2}$

$\Rightarrow \left(x-\frac{7}{4}\right)^2=\frac{25}{16}$

$\Rightarrow \left(x-\frac{7}{4}\right)=\pm\frac{5}{4}$

$\Rightarrow x=\frac{7}{4}\pm\frac{5}{4}$

$\Rightarrow x=\frac{7}{4}+\frac{5}{4}$ or $x=\frac{7}{4}-\frac{5}{4}$

$\Rightarrow x=\frac{12}{4}$ or $x=\frac{2}{4}$

$\Rightarrow x=3$ or $x=\frac{1}{2}$

ii. $2x^2+x-4=0$

Ans: $2x^2+x-4=0$

On dividing both sides of the equation by 2.

$\Rightarrow x^2 + \frac{1}{2}x = 2$

On adding $\left(\frac{1}{4}\right)^2$ both sides of equation.

$\Rightarrow x^2 + 2\left(\frac{1}{4}\right)x + \left(\frac{1}{4}\right)^2 = 2 + \left(\frac{1}{4}\right)^2$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{33}{16}$

$\Rightarrow \left(x + \frac{1}{4}\right) = \pm \frac{33}{4}$

$\Rightarrow x = \pm \frac{33}{4} - \frac{1}{4}$

$\Rightarrow x = \pm \frac{33 - 1}{4}$

$\Rightarrow x = \frac{33 - 1}{4}$ **or** $\Rightarrow x = \frac{33 + 1}{4}$

iii. $4x^2 + 4\ 3x + 3 = 0$

Ans: $4x^2 + 4\ 3x + 3 = 0$

$\Rightarrow (2x)^2 + 2(2\ 3)x + (\ 3)^2 = 0$

$\Rightarrow (2x + \ 3)^2 = 0$

$\Rightarrow (2x + \ 3) = 0$ and $\Rightarrow (2x + \ 3) = 0$

$\Rightarrow x = \frac{-\ 3}{2}$ and $\Rightarrow x = \frac{-\ 3}{2}$

iv. $2x^2 + x + 4 = 0$

Ans: $2x^2 + x + 4 = 0$

On dividing both sides of the equation by 2.

$\Rightarrow x^2 + \frac{1}{2}x + 2 = 0$

$\Rightarrow x^2 + 2\left(\frac{1}{4}\right)x = -2$

On adding $\left(\frac{1}{4}\right)^2$ both sides of equation.

$\Rightarrow x^2+2\left(\frac{1}{4}\right)x+\left(\frac{1}{4}\right)^2=-2+\left(\frac{1}{4}\right)^2$

$\Rightarrow \left(x+\frac{1}{4}\right)^2=\frac{1}{16}-2$

$\Rightarrow \left(x+\frac{1}{4}\right)^2=-\frac{31}{16}$

Since, the square of a number cannot be negative.
Therefore, there is no real root for the given equation.

2. Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula.

i. $2x^2-7x+3=0$

Ans: $2x^2-7x+3=0$

On comparing this equation with $ax^2+bx+c=0$.
So, $a=2,\ b=-7,\ c=3$.
Therefore, by using quadratic formula-

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\Rightarrow x=\frac{7\pm\sqrt{49-24}}{4}$

$\Rightarrow x=\frac{7\pm5}{4}$

$\Rightarrow x=\frac{7+5}{4}$ or $\Rightarrow x=\frac{7-5}{4}$

$\Rightarrow x=\frac{12}{4}$ or $\Rightarrow x=\frac{2}{4}$

$\therefore x=3$ or $\frac{1}{2}$.

ii. $2x^2+x-4=0$

Ans: $2x^2+x-4=0$

On comparing this equation with $ax^2+bx+c=0$.
So, $a=2,\ b=1,\ c=-4$.
Therefore, by using quadratic formula-

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x=\frac{-1\pm\sqrt{1-32}}{4}$$

$$\Rightarrow x=\frac{-1\pm\ \ 33}{4}$$

$$\Rightarrow x=\frac{-1+\ \ 33}{4}\ \text{or}\ \Rightarrow x=\frac{-1-\ 33}{4}$$

$$\therefore x=\frac{-1+\ 33}{4}\ \text{or}\ \frac{-1-\ \ 33}{4}.$$

iii. $4x^2+4\ \ 3x+3=0$

Ans: $4x^2+4\ \ 3x+3=0$

On comparing this equation with $ax^2+bx+c=0$.

So, $a=4,\ b=4\ \ 3,\ c=3$.

Therefore, by using quadratic formula-

$$x=\frac{-b\pm\ \ b^2-4ac}{2a}$$

$$\Rightarrow x=\frac{-4\ \ 3\pm\ \ 48-48}{8}$$

$$\Rightarrow x=\frac{-4\ \ 3}{8}$$

$$\Rightarrow x=\frac{-\ \ 3}{2}\ \text{or}\ \Rightarrow x=\frac{-\ 3}{2}$$

$$\therefore x=\frac{-\ 3}{2}\ \text{or}\ \frac{-\ \ 3}{2}.$$

iv. $2x^2+x+4=0$

Ans: $2x^2+x+4=0$

On comparing this equation with $ax^2+bx+c=0$.

So, $a=2,\ b=1,\ c=4$.

Therefore, by using quadratic formula-

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$\Rightarrow x=\dfrac{-1\pm\sqrt{1-32}}{4}$

$\Rightarrow x=\dfrac{-1\pm\sqrt{-31}}{4}$

Since, the square of a number cannot be negative.
Therefore, there is no real root for the given equation.

3. Find the roots of the following equations:

i. $x-\dfrac{1}{x}=3, x\neq 0$

Ans: $x-\dfrac{1}{x}=3, x\neq 0$

On comparing this equation with $ax^2+bx+c=0$.
So, $a=1,\ b=-3,\ c=-1$.
Therefore, by using quadratic formula-

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$\Rightarrow x=\dfrac{3\pm\sqrt{9+4}}{2}$

$\Rightarrow x=\dfrac{3\pm\sqrt{13}}{2}$

$\Rightarrow x=\dfrac{3+\sqrt{13}}{2}$ or $\Rightarrow x=\dfrac{3-\sqrt{13}}{2}$

$\therefore x=\dfrac{3+\sqrt{13}}{2}$ or $\dfrac{3-\sqrt{13}}{2}$.

ii. $\dfrac{1}{x+4}-\dfrac{1}{x-7}=\dfrac{11}{30}, x\neq -4,7$

Ans: $\dfrac{1}{x+4}-\dfrac{1}{x-7}=\dfrac{11}{30}, x\neq -4,7$

$\Rightarrow \dfrac{x-7-x-4}{(x+4)(x-7)}=\dfrac{11}{30}$

$\Rightarrow \dfrac{-11}{(x+4)(x-7)}=\dfrac{11}{30}$

$\Rightarrow (x+4)(x-7)=-30$

$\Rightarrow x^2-3x-28=-30$

$\Rightarrow x^2-3x+2=0$
$\Rightarrow x^2-2x-x+2=0$
$\Rightarrow x(x-2)-1(x-2)=0$
$\Rightarrow (x-2)(x-1)=0$
$\Rightarrow x=1 \text{ or } 2$

4. The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Ans: Let the present age of Rehman be x years.
Three years ago, his age is $(x-3)$ years.
Five years hence, his age will be $(x+5)$ years.
Therefore, $\frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$
$\Rightarrow \frac{x+5+x-3}{(x-3)(x+5)}=\frac{1}{3}$
$\Rightarrow \frac{2x+2}{(x-3)(x+5)}=\frac{1}{3}$
$\Rightarrow 3(2x+2)=(x-3)(x+5)$
$\Rightarrow 6x+6=x^2+2x-15$
$\Rightarrow x^2-4x-21=0$
$\Rightarrow x^2-7x+3x-21=0$
$\Rightarrow x(x-7)+3(x-7)=0$
$\Rightarrow (x-7)(x+3)=0$
$\Rightarrow x=7 \text{ or } -3$
Therefore, Rehman's age is 7 years.

5. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Ans: Let the marks in maths be x.
Thus, marks in English will be $30-x$.
Hence, according to question –
$(x+2)(30-x-3)=210$
$(x+2)(27-x)=210$

$\Rightarrow -x^2+25x+54=210$
$\Rightarrow x^2-25x+156=0$
$\Rightarrow x^2-12x-13x+156=0$
$\Rightarrow x(x-12)-13(x-12)=0$
$\Rightarrow (x-12)(x-13)=0$
$\Rightarrow x=12,13$
Case 1- If the marks in mathematics are 12 , then marks in English will be 18.
Case 2- If the marks in mathematics are 13 , then marks in English will be 17.

6. The diagonal of a rectangular field is 60metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.
Ans: Let the shorter side of the rectangle be x m.
Thus, Larger side of the rectangle will be $(x+30)$m .

Diagonal of the rectangle be $x^2+(x+30)^2$

Hence, according to question-

$x^2+(x+30)^2=x+60$

$\Rightarrow x^2+(x+30)^2=(x+60)^2$
$\Rightarrow x^2+x^2+900+60x=x^2+3600+120x$
$\Rightarrow x^2-60x-2700=0$
$\Rightarrow x^2-90x+30-2700=0$
$\Rightarrow x(x-90)+30(x-90)=0$
$\Rightarrow (x-90)(x+30)=0$
$\Rightarrow x=90,-30$
Since, side cannot be negative.
Therefore, the length of the shorter side of rectangle is 90 m.
Hence, length of the larger side of the rectangle be 120 m.

7. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.
Ans: Let the larger number be x and smaller number be y.
According to question-
$x^2-y^2=180$ and $y^2=8x$
$\Rightarrow x^2-8x=180$
$\Rightarrow x^2-8x-180=0$

$\Rightarrow x^2-18x-10x-180=0$

$\Rightarrow x(x-18)+10(x-18)=0$

$\Rightarrow (x-18)(x+10)=0$

$\Rightarrow x=18,-10$

Since, larger cannot be negative as 8 times of the larger number will be negative and hence, the square of the smaller number will be negative which is not possible.

Therefore, the larger number will be 18.

$\therefore y^2=8(18)$

$\Rightarrow y^2=144$

$\Rightarrow y=\pm 12$

Hence, smaller number be ± 12.

Therefore, the numbers are 18 and 12 or 18 and -12.

8. A train travels 360 km at a uniform speed. If the speed had been 5km/h more, it would have taken 1hour less for the same journey. Find the speed of the train.

Ans: Let the speed of the train be x km/h.

Time taken to cover 360 km/h be $\frac{360}{x}$.

According to question-

$(x+5)\left(\frac{360}{x}-1\right)=360$

$\Rightarrow 360-x+\frac{1800}{x}-5=360$

$\Rightarrow x^2+5x-1800=0$

$\Rightarrow x^2+45x-40x-1800=0$

$\Rightarrow x(x+45)-40(x+45)=0$

$\Rightarrow (x+45)(x-40)=0$

$\Rightarrow x=40,-45$

Since, the speed cannot be negative.

Therefore, the speed of the train is 40 km/h.

9. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Ans: Let the time taken by the smaller pipe to fill the tank be x hr.
So, time taken by larger pipe be $(x-10)$hr.
Part of the tank filled by smaller pipe in 1 hour is $\frac{1}{x}$.
Part of the tank filled by larger pipe in 1 hour is $\frac{1}{x-10}$.
So, according to question-

$$\frac{1}{x}+\frac{1}{x-10}=9\frac{3}{8}$$
$$\Rightarrow \frac{1}{x}+\frac{1}{x-10}=\frac{75}{8}$$
$$\Rightarrow \frac{x-10+x}{x(x-10)}=\frac{8}{75}$$
$$\Rightarrow \frac{2x-10}{x(x-10)}=\frac{8}{75}$$
$$\Rightarrow 75(2x-10)=8x^2-80x$$
$$\Rightarrow 150x-750=8x^2-80x$$
$$\Rightarrow 8x^2-230x+750=0\ 25$$
$$\Rightarrow 8x^2-200x-30x+750=0$$
$$\Rightarrow 8x(x-25)-30(x-25)=0$$
$$\Rightarrow (x-25)(8x-30)=0$$
$$\Rightarrow x=25 \text{ or } \frac{30}{8}$$

Case 1- If time taken by smaller pipe be $\frac{30}{8}$ i.e 3.75 hours.So, Time taken by larger pipe will be negative which is not possible.Hence, this case is rejected.
Case 2- If time taken by smaller pipe be .Then, time taken by larger pipe will be 15 hour.Therefore, time taken by smaller pipe be 25 and time taken by larger pipe will be 15 hour.

10. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speeds of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.
Ans: Let the average speed of passenger train be x km/h.

So, Average speed of express train be $(x+11)$km/h.

Thus, according to question.

$$\therefore \frac{132}{x}-\frac{132}{x+11}=1$$

$$\Rightarrow 132\left[\frac{x+11-x}{x(x+11)}\right]=1$$

$$\Rightarrow \frac{132\times 11}{x(x+11)}=1$$

$$\Rightarrow 132\times 11=x(x+11)$$

$$\Rightarrow x^2+11x-1452=0$$

$$\Rightarrow x^2+44x-33x-1452=0$$

$$\Rightarrow x(x+44)-33(x+44)=0$$

$$\Rightarrow (x+44)(x-33)=0$$

$$\Rightarrow x=-44 \text{ or } 33$$

Since, speed cannot be negative.

Therefore, the speed of the passenger train will be 33 km/h and thus, the speed of the express train will be 44 km/h .

11. Sum of the areas of two squares is 468 m^2. If the difference of their perimeters are 24 m, find the sides of the two squares.

Ans: Let the sides of the two squares be x m and y m.

Thus, their perimeters will be $4x$ and $4y$ and areas will be x^2 and y^2.

Hence, according to question –

$$4x-4y=24$$

$$\Rightarrow x-y=6$$

$$\Rightarrow x=y+6$$

And $x^2+y^2=468$

Substituting value of x-

$$(y+6)^2+y^2=468$$

$$\Rightarrow 36+y^2+12y+y^2=468$$

$$\Rightarrow 2y^2+12y-432=0$$

$$\Rightarrow y^2+6y-216=0$$

$$\Rightarrow y^2+18y-12y-216=0$$

$$\Rightarrow y(y+18)-12(y+18)=0$$

$\Rightarrow (y+18)(y-12)=0$

$\Rightarrow y=-18$ or 12

Since, side cannot be negative.

Therefore, the sides of the square are 12 m and $(12+6)\text{m}$ i.e 18 m.

Exercise 4.4

1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them-

i. $2x^2-3x+5=0$

Ans: For a quadratic equation $ax^2+bx+c=0$.

Where Discriminant $=b^2-4ac$

Then –

Case 1- If $b^2-4ac>0$ then there will be two distinct real roots.

Case 2- If $b^2-4ac=0$ then there will be two equal real roots.

Case 3- If $b^2-4ac<0$ then there will be no real roots.

Thus, for $2x^2-3x+5=0$ **.**

On comparing this equation with $ax^2+bx+c=0$.

So, $a=2,\ b=-3,\ c=5$.

Discriminant $=(-3)^2-4(2)(5)$

$=9-40$

$=-31$

Since, Discriminant: $b^2-4ac<0$.

Therefore, there is no real root for the given equation.

ii. $3x^2-4\ \ 3x+4=0$

Ans: For a quadratic equation $ax^2+bx+c=0$.

Where Discriminant $=b^2-4ac$

Then –

Case 1- If $b^2-4ac>0$ then there will be two distinct real roots.

Case 2- If $b^2-4ac=0$ then there will be two equal real roots.

Case 3- If $b^2-4ac<0$ then there will be no real roots.

Thus, for $3x^2-4\sqrt{3}x+4=0$ **.**

On comparing this equation with $ax^2+bx+c=0$.

So, $a=3,\ b=-4\sqrt{3},\ c=4$.

Discriminant $=\left(-4\sqrt{3}\right)^2-4\left(3\right)\left(4\right)$

$=48-48$

$=0$

Since, Discriminant: $b^2-4ac=0$.

Therefore, there is equal real root for the given equation and the roots are-

$\frac{-b}{2a}$ and $\frac{-b}{2a}$.

Hence, roots are-

$\frac{-b}{2a}=\frac{-\left(-4\quad 3\right)}{6}$

$=\frac{4\quad 3}{6}$

$=\frac{2\quad 3}{3}$

Therefore, roots are $\frac{2\quad 3}{3}$ and $\frac{2\ 3}{3}$.

iii. $2x^2$-6x+3=0

Ans: For a quadratic equation $ax^2+bx+c=0$.

Where Discriminant $=b^2-4ac$

Then –

Case 1- If $b^2-4ac>0$ then there will be two distinct real roots.

Case 2- If $b^2-4ac=0$ then there will be two equal real roots.

Case 3- If $b^2-4ac<0$ then there will be no real roots.

Thus, for $2x^2-6x+3=0$ **.**

On comparing this equation with $ax^2+bx+c=0$.

So, $a=2$, $b=-6$, $c=3$.

Discriminant $=\left(-6\right)^2-4\left(2\right)\left(3\right)$

$=36-24$

$=12$

Since, Discriminant: $b^2-4ac>0$.

Therefore, distinct real roots exists for the given equation and the roots are-

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Hence, roots are-

$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(2)(3)}}{4}$$

$$=\frac{6\pm\sqrt{36-24}}{4}$$

$$=\frac{6\pm\ \ 12}{4}$$

$$=\frac{6\pm2\ \ 3}{4}$$

$$=\frac{3\pm\ \ 3}{2}$$

Therefore, roots are $\frac{3+\ \ 3}{2}$ and $\frac{3-\ \ 3}{2}$.

2. Find the values of k for each of the following quadratic equations, so that they have two equal roots.

i. $2x^2+kx+3=0$

Ans: If a quadratic equation $ax^2+bx+c=0$ has two equal roots, then its discriminant will be 0 i.e., $b^2-4ac=0$

So, for $2x^2+kx+3=0$.

On comparing this equation with $ax^2+bx+c=0$.

So, $a=2$, $b=k$, $c=3$.

Discriminant $=(k)^2-4(2)(3)$

$=k^2-24$

For equal roots-

$b^2-4ac=0$

$\therefore k^2-24=0$

$\Rightarrow k^2=24$

$\Rightarrow k=\ \ 24$

$\Rightarrow k=\pm2\ \ 6$

ii. $kx(x-2)+6=0$

Ans: If a quadratic equation $ax^2+bx+c=0$ has two equal roots, then its discriminant will be 0 i.e., $b^2-4ac=0$

So, for $kx(x-2)+6=0$

$\Rightarrow kx^2-2kx+6=0$
On comparing this equation with $ax^2+bx+c=0$.
So, $a=k$, $b=-2k$, $c=6$.
Discriminant $=(-2k)^2-4(k)(6)$
$=4k^2-24k$
For equal roots-
$b^2-4ac=0$
$\therefore 4k^2-24k=0$
$\Rightarrow 4k(k-6)=0$
$\Rightarrow k=0$ or $k=6$
But k cannot be zero. Thus, this equation has two equal roots when k should be 6.

3. Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is $800m^2$? If so, find its length and breadth.
Ans: Let the breadth of mango grove be x.
So, length of mango grove will be $2x$.
Hence, Area of mango grove is $=(2x)x$
$=2x^2$.
So, $2x^2=800$
$\Rightarrow x^2=400$
$\Rightarrow x^2-400=0$
On comparing this equation with $ax^2+bx+c=0$.
So, $a=1$, $b=0$, $c=400$.
Discriminant $=(0)^2-4(1)(-400)$
$=1600$
Since, Discriminant: $b^2-4ac>0$.
Therefore, distinct real roots exist for the given equation and the roots are-

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Hence, roots are-

$$x=\frac{-(0)\pm\sqrt{(0)^2-4(1)(-400)}}{2}$$

$$=\frac{\pm\sqrt{1600}}{2}$$

$= \frac{\pm 40}{2}$

$= \pm 20$

Since, length cannot be negative.
Therefore, breadth of the mango grove is 20m.

And length of the mango grove be $2(20)\text{m}$ i.e., 40m.

4. Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Ans: Let the age of one friend be x years.

So, age of the other friend will be $(20\text{-}x)$ years.

Thus, four years ago, the age of one friend be $(x\text{-}4)$ years.

And age of the other friend will be $(16\text{-}x)$ years.

Hence, according to question-

$(x-4)(16-x)=48$

$\Rightarrow 16x-64-x^2+4x=48$

$\Rightarrow 20x-112-x^2=0$

$\Rightarrow x^2-20x+112-=0$

On comparing this equation with $ax^2+bx+c=0$.

So, $a=1$, $b=-20$, $c=112$.

Discriminant $=(-20)^2-4(1)(112)$

$=400-448$

$=-48$

Since, Discriminant: $b^2-4ac<0$.

Therefore, there is no real root for the given equation and hence, this situation is not possible.

5. Is it possible to design a rectangular park of perimeter 80 mand area 400m^2? If so find its length and breadth.

Ans: Let the length of the park be x m and breadth of the park be x m.

Thus, $\text{Perimeter}=2(x+y)$.

Hence, according to question-

$2(x+y)=80$

$\Rightarrow x+y=40$

$\Rightarrow y=40-x$.

Now, $\text{Area}=x\times y$.

Substituting value of y.

$\text{Area}=x(40-x)$

So, according to question-

$x(40-x)=400$

$\Rightarrow x^2-40x+400=0$

On comparing this equation with $ax^2+bx+c=0$.

So, $a=1$, $b=-40$, $c=400$.

Discriminant $=(-40)^2-4(1)(400)$

$=1600-1600$

$=0$

Since, Discriminant: $b^2-4ac=0$.

Therefore, there is equal real roots for the given equation and hence, this situation is possible.

Hence, roots are-

$\frac{-b}{2a}=\frac{-(-40)}{2}$

$=\frac{40}{2}$

$=20$

Therefore, length of park is $x=20\text{m}$.

And breadth of park be $y=(40-20)\text{m}$ i.e., $y=20\text{m}$.