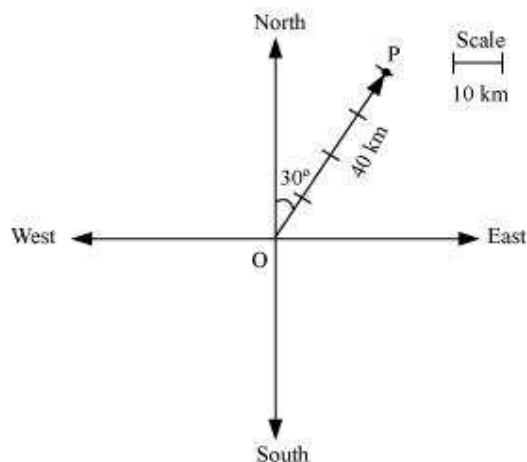


## Exercise 10.1

**Question 1:**

Represent graphically a displacement of 40 km,  $30^\circ$  east of north.

Answer



Here, vector  $\vec{OP}$  represents the displacement of 40 km,  $30^\circ$  East of North.

**Question 2:**

Classify the following measures as scalars and vectors.

- (i) 10 kg (ii) 2 metres north-west (iii)  $40^\circ$   
(iv) 40 watt (v)  $10^{-19}$  coulomb (vi)  $20 \text{ m/s}^2$

Answer

- (i) 10 kg is a scalar quantity because it involves only magnitude.  
(ii) 2 meters north-west is a vector quantity as it involves both magnitude and direction.  
(iii)  $40^\circ$  is a scalar quantity as it involves only magnitude.  
(iv) 40 watts is a scalar quantity as it involves only magnitude.  
(v)  $10^{-19}$  coulomb is a scalar quantity as it involves only magnitude.  
(vi)  $20 \text{ m/s}^2$  is a vector quantity as it involves magnitude as well as direction.

**Question 3:**

Classify the following as scalar and vector quantities.

- (i) time period (ii) distance (iii) force

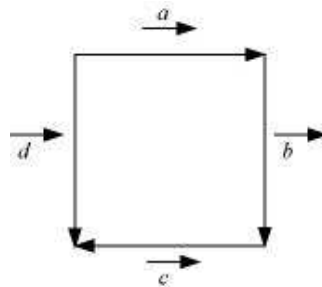
(iv) velocity (v) work done

Answer

- (i) Time period is a scalar quantity as it involves only magnitude.
- (ii) Distance is a scalar quantity as it involves only magnitude.
- (iii) Force is a vector quantity as it involves both magnitude and direction.
- (iv) Velocity is a vector quantity as it involves both magnitude as well as direction.
- (v) Work done is a scalar quantity as it involves only magnitude.

**Question 4:**

In Figure, identify the following vectors.



(i) Coinitial (ii) Equal (iii) Collinear but not equal

Answer

- (i) Vectors  $\vec{a}$  and  $\vec{d}$  are coinitial because they have the same initial point.
- (ii) Vectors  $\vec{b}$  and  $\vec{d}$  are equal because they have the same magnitude and direction.
- (iii) Vectors  $\vec{a}$  and  $\vec{c}$  are collinear but not equal. This is because although they are parallel, their directions are not the same.

**Question 5:**

Answer the following as true or false.

- (i)  $\vec{a}$  and  $-\vec{a}$  are collinear.
- (ii) Two collinear vectors are always equal in magnitude.
- (iii) Two vectors having same magnitude are collinear.
- (iv) Two collinear vectors having the same magnitude are equal.

Answer

- (i) True.

Vectors  $\vec{a}$  and  $-\vec{a}$  are parallel to the same line.

(ii) False.

Collinear vectors are those vectors that are parallel to the same line.

(iii) False.

## Exercise 10.2

**Question 1:**

Compute the magnitude of the following vectors:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}; \quad \vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}; \quad \vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$$

Answer

The given vectors are:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}; \quad \vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}; \quad \vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$$

$$|\vec{a}| = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3}$$

$$\begin{aligned} |\vec{b}| &= \sqrt{(2)^2 + (-7)^2 + (-3)^2} \\ &= \sqrt{4 + 49 + 9} \\ &= \sqrt{62} \end{aligned}$$

$$\begin{aligned} |\vec{c}| &= \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(-\frac{1}{\sqrt{3}}\right)^2} \\ &= \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = 1 \end{aligned}$$

**Question 2:**

Write two different vectors having same magnitude.

Answer

$$\text{Consider } \vec{a} = (\hat{i} - 2\hat{j} + 3\hat{k}) \text{ and } \vec{b} = (2\hat{i} + \hat{j} - 3\hat{k}).$$

$$\text{It can be observed that } |\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14} \text{ and}$$

$$|\vec{b}| = \sqrt{2^2 + 1^2 + (-3)^2} = \sqrt{4 + 1 + 9} = \sqrt{14}.$$

Hence,  $\vec{a}$  and  $\vec{b}$  are two different vectors having the same magnitude. The vectors are different because they have different directions.

**Question 3:**

Write two different vectors having same direction.

Answer

Consider  $\vec{p} = (\hat{i} + \hat{j} + \hat{k})$  and  $\vec{q} = (2\hat{i} + 2\hat{j} + 2\hat{k})$ .

The direction cosines of  $\vec{p}$  are given by,

$$l = \frac{1}{\sqrt{1^2+1^2+1^2}} = \frac{1}{\sqrt{3}}, \quad m = \frac{1}{\sqrt{1^2+1^2+1^2}} = \frac{1}{\sqrt{3}}, \quad \text{and } n = \frac{1}{\sqrt{1^2+1^2+1^2}} = \frac{1}{\sqrt{3}}.$$

The direction cosines of  $\vec{q}$  are given by

$$l = \frac{2}{\sqrt{2^2+2^2+2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}, \quad m = \frac{2}{\sqrt{2^2+2^2+2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}},$$

$$\text{and } n = \frac{2}{\sqrt{2^2+2^2+2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}.$$

The direction cosines of  $\vec{p}$  and  $\vec{q}$  are the same. Hence, the two vectors have the same direction.

#### Question 4:

Find the values of  $x$  and  $y$  so that the vectors  $2\hat{i} + 3\hat{j}$  and  $x\hat{i} + y\hat{j}$  are equal

Answer

The two vectors  $2\hat{i} + 3\hat{j}$  and  $x\hat{i} + y\hat{j}$  will be equal if their corresponding components are equal.

Hence, the required values of  $x$  and  $y$  are 2 and 3 respectively.

#### Question 5:

Find the scalar and vector components of the vector with initial point (2, 1) and terminal point (-5, 7).

Answer

The vector with the initial point P (2, 1) and terminal point Q (-5, 7) can be given by,

$$\vec{PQ} = (-5-2)\hat{i} + (7-1)\hat{j}$$

$$\Rightarrow \vec{PQ} = -7\hat{i} + 6\hat{j}$$

Hence, the required scalar components are -7 and 6 while the vector components are  $-7\hat{i}$  and  $6\hat{j}$ .

**Question 6:**

Find the sum of the vectors  $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ ,  $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$  and  $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$ .

Answer

The given vectors are  $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ ,  $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$  and  $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$ .

$$\begin{aligned}\therefore \vec{a} + \vec{b} + \vec{c} &= (1-2+1)\hat{i} + (-2+4-6)\hat{j} + (1+5-7)\hat{k} \\ &= 0\hat{i} - 4\hat{j} - 1\hat{k} \\ &= -4\hat{j} - \hat{k}\end{aligned}$$

**Question 7:**

Find the unit vector in the direction of the vector  $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ .

Answer

The unit vector  $\hat{a}$  in the direction of vector  $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$  is given by  $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$ .

$$|\vec{a}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{1+1+4} = \sqrt{6}$$

$$\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$$

**Question 8:**

Find the unit vector in the direction of vector  $\overline{PQ}$ , where P and Q are the points (1, 2, 3) and (4, 5, 6), respectively.

Answer

The given points are P (1, 2, 3) and Q (4, 5, 6).

$$\therefore \overline{PQ} = (4-1)\hat{i} + (5-2)\hat{j} + (6-3)\hat{k} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

$$|\overline{PQ}| = \sqrt{3^2 + 3^2 + 3^2} = \sqrt{9+9+9} = \sqrt{27} = 3\sqrt{3}$$

Hence, the unit vector in the direction of  $\overline{PQ}$  is

$$\frac{\overline{PQ}}{|\overline{PQ}|} = \frac{3\hat{i} + 3\hat{j} + 3\hat{k}}{3\sqrt{3}} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

**Question 9:**

For given vectors,  $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$  and  $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$ , find the unit vector in the direction of the vector  $\vec{a} + \vec{b}$

Answer

The given vectors are  $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$  and  $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$ .

$$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{b} = -\hat{i} + \hat{j} - \hat{k}$$

$$\therefore \vec{a} + \vec{b} = (2-1)\hat{i} + (-1+1)\hat{j} + (2-1)\hat{k} = 1\hat{i} + 0\hat{j} + 1\hat{k} = \hat{i} + \hat{k}$$

$$|\vec{a} + \vec{b}| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

Hence, the unit vector in the direction of  $(\vec{a} + \vec{b})$  is

$$\frac{(\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|} = \frac{\hat{i} + \hat{k}}{\sqrt{2}} = \frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$$

**Question 10:**

Find a vector in the direction of vector  $5\hat{i} - \hat{j} + 2\hat{k}$  which has magnitude 8 units.

Answer

$$\text{Let } \vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}.$$

$$\therefore |\vec{a}| = \sqrt{5^2 + (-1)^2 + 2^2} = \sqrt{25 + 1 + 4} = \sqrt{30}$$

$$\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}}$$

Hence, the vector in the direction of vector  $5\hat{i} - \hat{j} + 2\hat{k}$  which has magnitude 8 units is given by,

$$8\hat{a} = 8 \left( \frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}} \right) = \frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$$

$$= 8 \left( \frac{5\vec{i} - \vec{j} + 2\vec{k}}{\sqrt{30}} \right)$$

$$= \frac{40}{\sqrt{30}}\vec{i} - \frac{8}{\sqrt{30}}\vec{j} + \frac{16}{\sqrt{30}}\vec{k}$$

**Question 11:**

Show that the vectors  $2\hat{i} - 3\hat{j} + 4\hat{k}$  and  $-4\hat{i} + 6\hat{j} - 8\hat{k}$  are collinear.

Answer

Let  $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$  and  $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k}$ .

It is observed that  $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k} = -2(2\hat{i} - 3\hat{j} + 4\hat{k}) = -2\vec{a}$

$$\therefore \vec{b} = \lambda\vec{a}$$

where,

$$\lambda = -2$$

Hence, the given vectors are collinear.

**Question 12:**

Find the direction cosines of the vector  $\hat{i} + 2\hat{j} + 3\hat{k}$

Answer

Let  $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ .

$$\therefore |\vec{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$$

Hence, the direction cosines of  $\vec{a}$  are  $\left( \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right)$ .

**Question 13:**

Find the direction cosines of the vector joining the points A (1, 2, -3) and B (-1, -2, 1) directed from A to B.

Answer

The given points are A (1, 2, -3) and B (-1, -2, 1).



$$\therefore \overline{AB} = (-1-1)\hat{i} + (-2-2)\hat{j} + \{1-(-3)\}\hat{k}$$

$$\Rightarrow \overline{AB} = -2\hat{i} - 4\hat{j} + 4\hat{k}$$

$$\therefore |\overline{AB}| = \sqrt{(-2)^2 + (-4)^2 + 4^2} = \sqrt{4+16+16} = \sqrt{36} = 6$$

Hence, the direction cosines of  $\overline{AB}$  are  $\left(-\frac{2}{6}, -\frac{4}{6}, \frac{4}{6}\right) = \left(-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right)$ .

**Question 14:**

Show that the vector  $\hat{i} + \hat{j} + \hat{k}$  is equally inclined to the axes OX, OY, and OZ.

Answer

Let  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ .

Then,

$$|\vec{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

Therefore, the direction cosines of  $\vec{a}$  are  $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ .

Now, let  $\alpha$ ,  $\beta$ , and  $\gamma$  be the angles formed by  $\vec{a}$  with the positive directions of  $x$ ,  $y$ , and  $z$  axes.

Then, we have  $\cos \alpha = \frac{1}{\sqrt{3}}$ ,  $\cos \beta = \frac{1}{\sqrt{3}}$ ,  $\cos \gamma = \frac{1}{\sqrt{3}}$ .

Hence, the given vector is equally inclined to axes OX, OY, and OZ.

**Question 15:**

Find the position vector of a point R which divides the line joining two points P and Q

whose position vectors are  $\hat{i} + 2\hat{j} - \hat{k}$  and  $-\hat{i} + \hat{j} + \hat{k}$  respectively, in the ratio 2:1

(i) internally

(ii) externally

Answer

The position vector of point R dividing the line segment joining two points

P and Q in the ratio  $m:n$  is given by:

i. Internally:

$$\frac{m\vec{b} + n\vec{a}}{m + n}$$

ii. Externally:

$$\frac{m\vec{b} - n\vec{a}}{m - n}$$

Position vectors of P and Q are given as:

$$\overline{OP} = \hat{i} + 2\hat{j} - \hat{k} \text{ and } \overline{OQ} = -\hat{i} + \hat{j} + \hat{k}$$

(i) The position vector of point R which divides the line joining two points P and Q internally in the ratio 2:1 is given by,

$$\begin{aligned} \overline{OR} &= \frac{2(-\hat{i} + \hat{j} + \hat{k}) + 1(\hat{i} + 2\hat{j} - \hat{k})}{2+1} = \frac{(-2\hat{i} + 2\hat{j} + 2\hat{k}) + (\hat{i} + 2\hat{j} - \hat{k})}{3} \\ &= \frac{-\hat{i} + 4\hat{j} + \hat{k}}{3} = -\frac{1}{3}\hat{i} + \frac{4}{3}\hat{j} + \frac{1}{3}\hat{k} \end{aligned}$$

(ii) The position vector of point R which divides the line joining two points P and Q externally in the ratio 2:1 is given by,

$$\begin{aligned} \overline{OR} &= \frac{2(-\hat{i} + \hat{j} + \hat{k}) - 1(\hat{i} + 2\hat{j} - \hat{k})}{2-1} = \frac{(-2\hat{i} + 2\hat{j} + 2\hat{k}) - (\hat{i} + 2\hat{j} - \hat{k})}{1} \\ &= -3\hat{i} + 3\hat{k} \end{aligned}$$

### Question 16:

Find the position vector of the mid point of the vector joining the points P (2, 3, 4) and Q (4, 1, - 2).

Answer

The position vector of mid-point R of the vector joining points P (2, 3, 4) and Q (4, 1, - 2) is given by,

$$\begin{aligned} \overline{OR} &= \frac{(2\hat{i} + 3\hat{j} + 4\hat{k}) + (4\hat{i} + \hat{j} - 2\hat{k})}{2} = \frac{(2+4)\hat{i} + (3+1)\hat{j} + (4-2)\hat{k}}{2} \\ &= \frac{6\hat{i} + 4\hat{j} + 2\hat{k}}{2} = 3\hat{i} + 2\hat{j} + \hat{k} \end{aligned}$$

**Question 17:**

Show that the points A, B and C with position vectors,  $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$ ,

$\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$  and  $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$ , respectively form the vertices of a right angled triangle.

Answer

Position vectors of points A, B, and C are respectively given as:

$$\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k} \text{ and } \vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$$

$$\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k} \text{ and } \vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$$

$$\therefore \overline{AB} = \vec{b} - \vec{a} = (2-3)\hat{i} + (-1+4)\hat{j} + (1+4)\hat{k} = -\hat{i} + 3\hat{j} + 5\hat{k}$$

$$\overline{BC} = \vec{c} - \vec{b} = (1-2)\hat{i} + (-3+1)\hat{j} + (-5-1)\hat{k} = -\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\overline{CA} = \vec{a} - \vec{c} = (3-1)\hat{i} + (-4+3)\hat{j} + (-4+5)\hat{k} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\therefore |\overline{AB}|^2 = (-1)^2 + 3^2 + 5^2 = 1 + 9 + 25 = 35$$

$$|\overline{BC}|^2 = (-1)^2 + (-2)^2 + (-6)^2 = 1 + 4 + 36 = 41$$

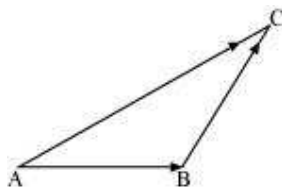
$$|\overline{CA}|^2 = 2^2 + (-1)^2 + 1^2 = 4 + 1 + 1 = 6$$

$$\therefore |\overline{AB}|^2 + |\overline{CA}|^2 = 35 + 6 = 41 = |\overline{BC}|^2$$

Hence, ABC is a right-angled triangle.

**Question 18:**

In triangle ABC which of the following is **not** true:



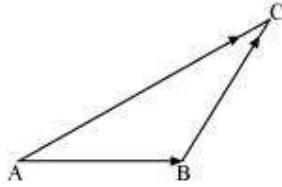
A.  $\overline{AB} + \overline{BC} + \overline{CA} = \vec{0}$

B.  $\overline{AB} + \overline{BC} - \overline{AC} = \vec{0}$

C.  $\overline{AB} + \overline{BC} - \overline{CA} = \vec{0}$

D.  $\overline{AB} - \overline{CB} + \overline{CA} = \vec{0}$

Answer



On applying the triangle law of addition in the given triangle, we have:

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} \quad \dots(1)$$

$$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} = -\overrightarrow{CA}$$

$$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0} \quad \dots(2)$$

$\therefore$  The equation given in alternative A is true.

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

$$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$$

$\therefore$  The equation given in alternative B is true.

From equation (2), we have:

$$\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$$

$\therefore$  The equation given in alternative D is true.

Now, consider the equation given in alternative C:

$$\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$$

$$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{CA} \quad \dots(3)$$

From equations (1) and (3), we have:

$$\overrightarrow{AC} = \overrightarrow{CA}$$

$$\Rightarrow \overrightarrow{AC} = -\overrightarrow{AC}$$

$$\Rightarrow \overrightarrow{AC} + \overrightarrow{AC} = \vec{0}$$

$$\Rightarrow 2\overrightarrow{AC} = \vec{0}$$

$$\Rightarrow \overrightarrow{AC} = \vec{0}, \text{ which is not true.}$$

Hence, the equation given in alternative C is **incorrect**.

The correct answer is **C**.

**Question 19:**

If  $\vec{a}$  and  $\vec{b}$  are two collinear vectors, then which of the following are **incorrect**:

- A.  $\vec{b} = \lambda\vec{a}$ , for some scalar  $\lambda$
- B.  $\vec{a} = \pm\vec{b}$
- C. the respective components of  $\vec{a}$  and  $\vec{b}$  are proportional
- D. both the vectors  $\vec{a}$  and  $\vec{b}$  have same direction, but different magnitudes

Answer

If  $\vec{a}$  and  $\vec{b}$  are two collinear vectors, then they are parallel.

Therefore, we have:

$$\vec{b} = \lambda\vec{a} \text{ (For some scalar } \lambda)$$

If  $\lambda = \pm 1$ , then  $\vec{a} = \pm\vec{b}$ .

If  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$  and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ , then

$$\vec{b} = \lambda\vec{a}.$$

$$\Rightarrow b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = \lambda(a_1\hat{i} + a_2\hat{j} + a_3\hat{k})$$

$$\Rightarrow b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = (\lambda a_1)\hat{i} + (\lambda a_2)\hat{j} + (\lambda a_3)\hat{k}$$

$$\Rightarrow b_1 = \lambda a_1, b_2 = \lambda a_2, b_3 = \lambda a_3$$

$$\Rightarrow \frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = \lambda$$

Thus, the respective components of  $\vec{a}$  and  $\vec{b}$  are proportional.

However, vectors  $\vec{a}$  and  $\vec{b}$  can have different directions.

Hence, the statement given in **D** is **incorrect**.

The correct answer is **D**.

## Exercise 10.3

**Question 1:**

Find the angle between two vectors  $\vec{a}$  and  $\vec{b}$  with magnitudes  $\sqrt{3}$  and 2, respectively

having  $\vec{a} \cdot \vec{b} = \sqrt{6}$ .

Answer

It is given that,

$$|\vec{a}| = \sqrt{3}, |\vec{b}| = 2 \text{ and, } \vec{a} \cdot \vec{b} = \sqrt{6}$$

Now, we know that  $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$ .

$$\therefore \sqrt{6} = \sqrt{3} \times 2 \times \cos\theta$$

$$\Rightarrow \cos\theta = \frac{\sqrt{6}}{\sqrt{3} \times 2}$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Hence, the angle between the given vectors  $\vec{a}$  and  $\vec{b}$  is  $\frac{\pi}{4}$ .

**Question 2:**

Find the angle between the vectors  $\hat{i} - 2\hat{j} + 3\hat{k}$  and  $3\hat{i} - 2\hat{j} + \hat{k}$

Answer

The given vectors are  $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$  and  $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$ .

$$|\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$$

$$|\vec{b}| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{9+4+1} = \sqrt{14}$$

$$\begin{aligned} \text{Now, } \vec{a} \cdot \vec{b} &= (\hat{i} - 2\hat{j} + 3\hat{k})(3\hat{i} - 2\hat{j} + \hat{k}) \\ &= 1.3 + (-2)(-2) + 3.1 \\ &= 3 + 4 + 3 \\ &= 10 \end{aligned}$$

Also, we know that  $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$ .

$$\therefore 10 = \sqrt{14}\sqrt{14}\cos\theta$$

$$\Rightarrow \cos\theta = \frac{10}{14}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{5}{7}\right)$$

### Question 3:

Find the projection of the vector  $\hat{i} - \hat{j}$  on the vector  $\hat{i} + \hat{j}$ .

Answer

Let  $\vec{a} = \hat{i} - \hat{j}$  and  $\vec{b} = \hat{i} + \hat{j}$ .

Now, projection of vector  $\vec{a}$  on  $\vec{b}$  is given by,

$$\frac{1}{|\vec{b}|}(\vec{a} \cdot \vec{b}) = \frac{1}{\sqrt{1+1}}\{1.1 + (-1)(1)\} = \frac{1}{\sqrt{2}}(1-1) = 0$$

Hence, the projection of vector  $\vec{a}$  on  $\vec{b}$  is 0.

### Question 4:

Find the projection of the vector  $\hat{i} + 3\hat{j} + 7\hat{k}$  on the vector  $7\hat{i} - \hat{j} + 8\hat{k}$ .

Answer

Let  $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$  and  $\vec{b} = 7\hat{i} - \hat{j} + 8\hat{k}$ .

Now, projection of vector  $\vec{a}$  on  $\vec{b}$  is given by,

$$\frac{1}{|\vec{b}|}(\vec{a} \cdot \vec{b}) = \frac{1}{\sqrt{7^2 + (-1)^2 + 8^2}} \{1(7) + 3(-1) + 7(8)\} = \frac{7-3+56}{\sqrt{49+1+64}} = \frac{60}{\sqrt{114}}$$

**Question 5:**

Show that each of the given three vectors is a unit vector:

$$\frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k}), \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k}), \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k})$$

Also, show that they are mutually perpendicular to each other.

Answer

$$\text{Let } \vec{a} = \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k}) = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k},$$

$$\vec{b} = \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k}) = \frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k},$$

$$\vec{c} = \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k}) = \frac{6}{7}\hat{i} + \frac{2}{7}\hat{j} - \frac{3}{7}\hat{k}.$$

$$|\vec{a}| = \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2} = \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}} = 1$$

$$|\vec{b}| = \sqrt{\left(\frac{3}{7}\right)^2 + \left(-\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2} = \sqrt{\frac{9}{49} + \frac{36}{49} + \frac{4}{49}} = 1$$

$$|\vec{c}| = \sqrt{\left(\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2 + \left(-\frac{3}{7}\right)^2} = \sqrt{\frac{36}{49} + \frac{4}{49} + \frac{9}{49}} = 1$$

Thus, each of the given three vectors is a unit vector.

$$\vec{a} \cdot \vec{b} = \frac{2}{7} \times \frac{3}{7} + \frac{3}{7} \times \left(-\frac{6}{7}\right) + \frac{6}{7} \times \frac{2}{7} = \frac{6}{49} - \frac{18}{49} + \frac{12}{49} = 0$$

$$\vec{b} \cdot \vec{c} = \frac{3}{7} \times \frac{6}{7} + \left(-\frac{6}{7}\right) \times \frac{2}{7} + \frac{2}{7} \times \left(-\frac{3}{7}\right) = \frac{18}{49} - \frac{12}{49} - \frac{6}{49} = 0$$

$$\vec{c} \cdot \vec{a} = \frac{6}{7} \times \frac{2}{7} + \frac{2}{7} \times \frac{3}{7} + \left(-\frac{3}{7}\right) \times \frac{6}{7} = \frac{12}{49} + \frac{6}{49} - \frac{18}{49} = 0$$

Hence, the given three vectors are mutually perpendicular to each other.



**Question 6:**

Find  $|\vec{a}|$  and  $|\vec{b}|$ , if  $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$  and  $|\vec{a}| = 8|\vec{b}|$ .

Answer

$$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$$

$$\Rightarrow \vec{a} \cdot \vec{a} - \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} - \vec{b} \cdot \vec{b} = 8$$

$$\Rightarrow |\vec{a}|^2 - |\vec{b}|^2 = 8$$

$$\Rightarrow (8|\vec{b}|)^2 - |\vec{b}|^2 = 8 \quad [|\vec{a}| = 8|\vec{b}|]$$

$$\Rightarrow 64|\vec{b}|^2 - |\vec{b}|^2 = 8$$

$$\Rightarrow 63|\vec{b}|^2 = 8$$

$$\Rightarrow |\vec{b}|^2 = \frac{8}{63}$$

$$\Rightarrow |\vec{b}| = \sqrt{\frac{8}{63}} \quad [\text{Magnitude of a vector is non-negative}]$$

$$\Rightarrow |\vec{b}| = \frac{2\sqrt{2}}{3\sqrt{7}}$$

$$|\vec{a}| = 8|\vec{b}| = \frac{8 \times 2\sqrt{2}}{3\sqrt{7}} = \frac{16\sqrt{2}}{3\sqrt{7}}$$

**Question 7:**

Evaluate the product  $(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b})$ .

Answer

$$(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b})$$

$$= 3\vec{a} \cdot 2\vec{a} + 3\vec{a} \cdot 7\vec{b} - 5\vec{b} \cdot 2\vec{a} - 5\vec{b} \cdot 7\vec{b}$$

$$= 6\vec{a} \cdot \vec{a} + 21\vec{a} \cdot \vec{b} - 10\vec{a} \cdot \vec{b} - 35\vec{b} \cdot \vec{b}$$

$$= 6|\vec{a}|^2 + 11\vec{a} \cdot \vec{b} - 35|\vec{b}|^2$$

**Question 8:**

Find the magnitude of two vectors  $\vec{a}$  and  $\vec{b}$ , having the same magnitude and such that

the angle between them is  $60^\circ$  and their scalar product is  $\frac{1}{2}$ .

Answer

Let  $\theta$  be the angle between the vectors  $\vec{a}$  and  $\vec{b}$ .

It is given that  $|\vec{a}| = |\vec{b}|$ ,  $\vec{a} \cdot \vec{b} = \frac{1}{2}$ , and  $\theta = 60^\circ$ . ... (1)

We know that  $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$ .

$$\therefore \frac{1}{2} = |\vec{a}||\vec{a}|\cos 60^\circ \quad [\text{Using (1)}]$$

$$\Rightarrow \frac{1}{2} = |\vec{a}|^2 \times \frac{1}{2}$$

$$\Rightarrow |\vec{a}|^2 = 1$$

$$\Rightarrow |\vec{a}| = |\vec{b}| = 1$$

**Question 9:**

Find  $|\vec{x}|$ , if for a unit vector  $\vec{a}$ ,  $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$ .

Answer

$$(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$$

$$\Rightarrow \vec{x} \cdot \vec{x} + \vec{x} \cdot \vec{a} - \vec{a} \cdot \vec{x} - \vec{a} \cdot \vec{a} = 12$$

$$\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 12$$

$$\Rightarrow |\vec{x}|^2 - 1 = 12 \quad [|\vec{a}| = 1 \text{ as } \vec{a} \text{ is a unit vector}]$$

$$\Rightarrow |\vec{x}|^2 = 13$$

$$\therefore |\vec{x}| = \sqrt{13}$$

**Question 10:**

If  $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$ ,  $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$  and  $\vec{c} = 3\hat{i} + \hat{j}$  are such that  $\vec{a} + \lambda\vec{b}$  is perpendicular to  $\vec{c}$ , then find the value of  $\lambda$ .

Answer

The given vectors are  $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$ ,  $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ , and  $\vec{c} = 3\hat{i} + \hat{j}$ .

Now,

$$\vec{a} + \lambda\vec{b} = (2\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + 2\hat{j} + \hat{k}) = (2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}$$

If  $(\vec{a} + \lambda\vec{b})$  is perpendicular to  $\vec{c}$ , then

$$(\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0.$$

$$\Rightarrow [(2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}] \cdot (3\hat{i} + \hat{j}) = 0$$

$$\Rightarrow (2 - \lambda)3 + (2 + 2\lambda)1 + (3 + \lambda)0 = 0$$

$$\Rightarrow 6 - 3\lambda + 2 + 2\lambda = 0$$

$$\Rightarrow -\lambda + 8 = 0$$

$$\Rightarrow \lambda = 8$$

Hence, the required value of  $\lambda$  is 8.

**Question 11:**

Show that  $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$  is perpendicular to  $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$ , for any two nonzero vectors  $\vec{a}$  and  $\vec{b}$

Answer

$$\begin{aligned} & (|\vec{a}|\vec{b} + |\vec{b}|\vec{a}) \cdot (|\vec{a}|\vec{b} - |\vec{b}|\vec{a}) \\ &= |\vec{a}|^2 \vec{b} \cdot \vec{b} - |\vec{a}||\vec{b}|\vec{b} \cdot \vec{a} + |\vec{b}||\vec{a}|\vec{a} \cdot \vec{b} - |\vec{b}|^2 \vec{a} \cdot \vec{a} \\ &= |\vec{a}|^2 |\vec{b}|^2 - |\vec{b}|^2 |\vec{a}|^2 \\ &= 0 \end{aligned}$$

Hence,  $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$  and  $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$  are perpendicular to each other.

**Question 12:**

If  $\vec{a} \cdot \vec{a} = 0$  and  $\vec{a} \cdot \vec{b} = 0$ , then what can be concluded about the vector  $\vec{b}$ ?

Answer

It is given that  $\vec{a} \cdot \vec{a} = 0$  and  $\vec{a} \cdot \vec{b} = 0$

Now,

$$\vec{a} \cdot \vec{a} = 0 \Rightarrow |\vec{a}|^2 = 0 \Rightarrow |\vec{a}| = 0$$

$\therefore \vec{a}$  is a zero vector.

Hence, vector  $\vec{b}$  satisfying  $\vec{a} \cdot \vec{b} = 0$  can be any vector.

**Question 14:**

If either vector  $\vec{a} = \vec{0}$  or  $\vec{b} = \vec{0}$ , then  $\vec{a} \cdot \vec{b} = 0$ . But the converse need not be true. Justify your answer with an example.

Answer

Consider  $\vec{a} = 2\hat{i} + 4\hat{j} + 3\hat{k}$  and  $\vec{b} = 3\hat{i} + 3\hat{j} - 6\hat{k}$ .

Then,

$$\vec{a} \cdot \vec{b} = 2 \cdot 3 + 4 \cdot 3 + 3(-6) = 6 + 12 - 18 = 0$$

We now observe that:

$$|\vec{a}| = \sqrt{2^2 + 4^2 + 3^2} = \sqrt{29}$$

$$\therefore \vec{a} \neq \vec{0}$$

$$|\vec{b}| = \sqrt{3^2 + 3^2 + (-6)^2} = \sqrt{54}$$

$$\therefore \vec{b} \neq \vec{0}$$

Hence, the converse of the given statement need not be true.

**Question 15:**

If the vertices A, B, C of a triangle ABC are (1, 2, 3), (-1, 0, 0), (0, 1, 2), respectively,

then find  $\sphericalangle$ ABC. [ $\sphericalangle$ ABC is the angle between the vectors  $\overrightarrow{BA}$  and  $\overrightarrow{BC}$ ]

Answer

The vertices of  $\triangle ABC$  are given as A (1, 2, 3), B (-1, 0, 0), and C (0, 1, 2).

Also, it is given that  $\sphericalangle$ ABC is the angle between the vectors  $\overrightarrow{BA}$  and  $\overrightarrow{BC}$ .

$$\overline{BA} = \{1 - (-1)\}\hat{i} + (2 - 0)\hat{j} + (3 - 0)\hat{k} = 2\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\overline{BC} = \{0 - (-1)\}\hat{i} + (1 - 0)\hat{j} + (2 - 0)\hat{k} = \hat{i} + \hat{j} + 2\hat{k}$$

$$\therefore \overline{BA} \cdot \overline{BC} = (2\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 2 \times 1 + 2 \times 1 + 3 \times 2 = 2 + 2 + 6 = 10$$

$$|\overline{BA}| = \sqrt{2^2 + 2^2 + 3^2} = \sqrt{4 + 4 + 9} = \sqrt{17}$$

$$|\overline{BC}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$$

Now, it is known that:

$$\overline{BA} \cdot \overline{BC} = |\overline{BA}| |\overline{BC}| \cos(\angle ABC)$$

$$\therefore 10 = \sqrt{17} \times \sqrt{6} \cos(\angle ABC)$$

$$\Rightarrow \cos(\angle ABC) = \frac{10}{\sqrt{17} \times \sqrt{6}}$$

$$\Rightarrow \angle ABC = \cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$$

#### Question 16:

Show that the points A (1, 2, 7), B (2, 6, 3) and C (3, 10, -1) are collinear.

Answer

The given points are A (1, 2, 7), B (2, 6, 3), and C (3, 10, -1).

$$\therefore \overline{AB} = (2 - 1)\hat{i} + (6 - 2)\hat{j} + (3 - 7)\hat{k} = \hat{i} + 4\hat{j} - 4\hat{k}$$

$$\overline{BC} = (3 - 2)\hat{i} + (10 - 6)\hat{j} + (-1 - 3)\hat{k} = \hat{i} + 4\hat{j} - 4\hat{k}$$

$$\overline{AC} = (3 - 1)\hat{i} + (10 - 2)\hat{j} + (-1 - 7)\hat{k} = 2\hat{i} + 8\hat{j} - 8\hat{k}$$

$$|\overline{AB}| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{1 + 16 + 16} = \sqrt{33}$$

$$|\overline{BC}| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{1 + 16 + 16} = \sqrt{33}$$

$$|\overline{AC}| = \sqrt{2^2 + 8^2 + (-8)^2} = \sqrt{4 + 64 + 64} = \sqrt{132} = 2\sqrt{33}$$

$$\therefore |\overline{AC}| = |\overline{AB}| + |\overline{BC}|$$

Hence, the given points A, B, and C are collinear.

**Question 17:**

Show that the vectors  $2\hat{i} - \hat{j} + \hat{k}$ ,  $\hat{i} - 3\hat{j} - 5\hat{k}$  and  $3\hat{i} - 4\hat{j} - 4\hat{k}$  form the vertices of a right angled triangle.

Answer

Let vectors  $2\hat{i} - \hat{j} + \hat{k}$ ,  $\hat{i} - 3\hat{j} - 5\hat{k}$  and  $3\hat{i} - 4\hat{j} - 4\hat{k}$  be position vectors of points A, B, and C respectively.

$$\text{i.e., } \overline{OA} = 2\hat{i} - \hat{j} + \hat{k}, \overline{OB} = \hat{i} - 3\hat{j} - 5\hat{k} \text{ and } \overline{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$$

Now, vectors  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{AC}$  represent the sides of  $\Delta ABC$ .

$$\text{i.e., } \overline{OA} = 2\hat{i} - \hat{j} + \hat{k}, \overline{OB} = \hat{i} - 3\hat{j} - 5\hat{k}, \text{ and } \overline{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$$

$$\therefore \overline{AB} = (1-2)\hat{i} + (-3+1)\hat{j} + (-5-1)\hat{k} = -\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\overline{BC} = (3-1)\hat{i} + (-4+3)\hat{j} + (-4+5)\hat{k} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\overline{AC} = (2-3)\hat{i} + (-1+4)\hat{j} + (1+4)\hat{k} = -\hat{i} + 3\hat{j} + 5\hat{k}$$

$$|\overline{AB}| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{1+4+36} = \sqrt{41}$$

$$|\overline{BC}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{4+1+1} = \sqrt{6}$$

$$|\overline{AC}| = \sqrt{(-1)^2 + 3^2 + 5^2} = \sqrt{1+9+25} = \sqrt{35}$$

$$\therefore |\overline{BC}|^2 + |\overline{AC}|^2 = 6 + 35 = 41 = |\overline{AB}|^2$$

Hence,  $\Delta ABC$  is a right-angled triangle.

**Question 18:**

If  $\vec{a}$  is a nonzero vector of magnitude 'a' and  $\lambda$  a nonzero scalar, then  $\lambda\vec{a}$  is unit vector if

$$(A) \lambda = 1 \quad (B) \lambda = -1 \quad (C) a = |\lambda| \quad (D) a = \frac{1}{|\lambda|}$$

Answer

Vector  $\lambda\vec{a}$  is a unit vector if  $|\lambda\vec{a}| = 1$ .

Now,

$$|\lambda\vec{a}| = 1$$

$$\Rightarrow |\lambda||\vec{a}| = 1$$

$$\Rightarrow |\vec{a}| = \frac{1}{|\lambda|} \quad [\lambda \neq 0]$$

$$\Rightarrow a = \frac{1}{|\lambda|} \quad [|\vec{a}| = a]$$

Hence, vector  $\lambda\vec{a}$  is a unit vector if  $a = \frac{1}{|\lambda|}$ .

The correct answer is D.

## Exercise 10.4

## Question 1:

Find  $|\vec{a} \times \vec{b}|$ , if  $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$  and  $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ .

Answer

We have,

$$\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k} \text{ and } \vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$$

$$\begin{aligned} \vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix} \\ &= \hat{i}(-14+14) - \hat{j}(2-21) + \hat{k}(-2+21) = 19\hat{j} + 19\hat{k} \\ \therefore |\vec{a} \times \vec{b}| &= \sqrt{(19)^2 + (19)^2} = \sqrt{2 \times (19)^2} = 19\sqrt{2} \end{aligned}$$

## Question 2:

Find a unit vector perpendicular to each of the vector  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$ , where

$$\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k} \text{ and } \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$$

Answer

We have,

$$\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k} \text{ and } \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$$

$$\therefore \vec{a} + \vec{b} = 4\hat{i} + 4\hat{j}, \quad \vec{a} - \vec{b} = 2\hat{i} + 4\hat{k}$$

$$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix} = \hat{i}(16) - \hat{j}(16) + \hat{k}(-8) = 16\hat{i} - 16\hat{j} - 8\hat{k}$$

$$\begin{aligned} \therefore |(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})| &= \sqrt{16^2 + (-16)^2 + (-8)^2} \\ &= \sqrt{2^2 \times 8^2 + 2^2 \times 8^2 + 8^2} \\ &= 8\sqrt{2^2 + 2^2 + 1} = 8\sqrt{9} = 8 \times 3 = 24 \end{aligned}$$



Hence, the unit vector perpendicular to each of the vectors  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$  is given by the relation,

$$\begin{aligned} &= \pm \frac{(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})}{|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})|} = \pm \frac{16\hat{i} - 16\hat{j} - 8\hat{k}}{24} \\ &= \pm \frac{2\hat{i} - 2\hat{j} - \hat{k}}{3} = \pm \frac{2}{3}\hat{i} \mp \frac{2}{3}\hat{j} \mp \frac{1}{3}\hat{k} \end{aligned}$$

### Question 3:

If a unit vector  $\vec{a}$  makes an angle  $\frac{\pi}{3}$  with  $\hat{i}$ ,  $\frac{\pi}{4}$  with  $\hat{j}$  and an acute angle  $\theta$  with  $\hat{k}$ , then find  $\theta$  and hence, the components of  $\vec{a}$ .

Answer

Let unit vector  $\vec{a}$  have  $(a_1, a_2, a_3)$  components.

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

Since  $\vec{a}$  is a unit vector,  $|\vec{a}| = 1$ .

Also, it is given that  $\vec{a}$  makes angles  $\frac{\pi}{3}$  with  $\hat{i}$ ,  $\frac{\pi}{4}$  with  $\hat{j}$ , and an acute angle  $\theta$  with  $\hat{k}$ .

Then, we have:

$$\cos \frac{\pi}{3} = \frac{a_1}{|\vec{a}|}$$

$$\Rightarrow \frac{1}{2} = a_1 \quad [|\vec{a}| = 1]$$

$$\cos \frac{\pi}{4} = \frac{a_2}{|\vec{a}|}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = a_2 \quad [|\vec{a}| = 1]$$

$$\text{Also, } \cos \theta = \frac{a_3}{|\vec{a}|}$$

$$\Rightarrow a_3 = \cos \theta$$

Now,

$$|a| = 1$$

$$\Rightarrow \sqrt{a_1^2 + a_2^2 + a_3^2} = 1$$

$$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2 \theta = 1$$

$$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2 \theta = 1$$

$$\Rightarrow \frac{3}{4} + \cos^2 \theta = 1$$

$$\Rightarrow \cos^2 \theta = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$\therefore a_3 = \cos \frac{\pi}{3} = \frac{1}{2}$$

Hence,  $\theta = \frac{\pi}{3}$  and the components of  $\vec{a}$  are  $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right)$ .

#### Question 4:

Show that

$$(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$$

Answer

$$(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$$

$$= (\vec{a} - \vec{b}) \times \vec{a} + (\vec{a} - \vec{b}) \times \vec{b}$$

[By distributivity of vector product over addition]

$$= \vec{a} \times \vec{a} - \vec{b} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{b}$$

[Again, by distributivity of vector product over addition]

$$= \vec{0} + \vec{a} \times \vec{b} + \vec{a} \times \vec{b} - \vec{0}$$

$$= 2\vec{a} \times \vec{b}$$

#### Question 5:

Find  $\lambda$  and  $\mu$  if  $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$ .

Answer

$$(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix} = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\Rightarrow \hat{i}(6\mu - 27\lambda) - \hat{j}(2\mu - 27) + \hat{k}(2\lambda - 6) = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

On comparing the corresponding components, we have:

$$6\mu - 27\lambda = 0$$

$$2\mu - 27 = 0$$

$$2\lambda - 6 = 0$$

Now,

$$2\lambda - 6 = 0 \Rightarrow \lambda = 3$$

$$2\mu - 27 = 0 \Rightarrow \mu = \frac{27}{2}$$

Hence,  $\lambda = 3$  and  $\mu = \frac{27}{2}$ .

### Question 6:

Given that  $\vec{a} \cdot \vec{b} = 0$  and  $\vec{a} \times \vec{b} = \vec{0}$ . What can you conclude about the vectors  $\vec{a}$  and  $\vec{b}$ ?

Answer

$$\vec{a} \cdot \vec{b} = 0$$

Then,

(i) Either  $|\vec{a}| = 0$  or  $|\vec{b}| = 0$ , or  $\vec{a} \perp \vec{b}$  (in case  $\vec{a}$  and  $\vec{b}$  are non-zero)

$$\vec{a} \times \vec{b} = \vec{0}$$

(ii) Either  $|\vec{a}| = 0$  or  $|\vec{b}| = 0$ , or  $\vec{a} \parallel \vec{b}$  (in case  $\vec{a}$  and  $\vec{b}$  are non-zero)

But,  $\vec{a}$  and  $\vec{b}$  cannot be perpendicular and parallel simultaneously.

Hence,  $|\vec{a}| = 0$  or  $|\vec{b}| = 0$ .

**Question 7:**

Let the vectors  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$  given as  $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ ,  $b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ ,  $c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ . Then show

$$\text{that } \vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

Answer

We have,

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}, \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

$$(\vec{b} + \vec{c}) = (b_1 + c_1)\hat{i} + (b_2 + c_2)\hat{j} + (b_3 + c_3)\hat{k}$$

$$\text{Now, } \vec{a} \times (\vec{b} + \vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 + c_1 & b_2 + c_2 & b_3 + c_3 \end{vmatrix}$$

$$\begin{aligned} &= \hat{i}[a_2(b_3 + c_3) - a_3(b_2 + c_2)] - \hat{j}[a_1(b_3 + c_3) - a_3(b_1 + c_1)] + \hat{k}[a_1(b_2 + c_2) - a_2(b_1 + c_1)] \\ &= \hat{i}[a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2] + \hat{j}[-a_1b_3 - a_1c_3 + a_3b_1 + a_3c_1] + \hat{k}[a_1b_2 + a_1c_2 - a_2b_1 - a_2c_1] \quad \dots(1) \end{aligned}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= \hat{i}[a_2b_3 - a_3b_2] + \hat{j}[b_1a_3 - a_1b_3] + \hat{k}[a_1b_2 - a_2b_1] \quad (2)$$

$$\vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \hat{i}[a_2c_3 - a_3c_2] + \hat{j}[a_3c_1 - a_1c_3] + \hat{k}[a_1c_2 - a_2c_1] \quad (3)$$

On adding (2) and (3), we get:

$$\begin{aligned} (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c}) &= \hat{i}[a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2] + \hat{j}[b_1a_3 + a_3c_1 - a_1b_3 - a_1c_3] \\ &+ \hat{k}[a_1b_2 + a_1c_2 - a_2b_1 - a_2c_1] \quad (4) \end{aligned}$$

Now, from (1) and (4), we have:

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

Hence, the given result is proved.

**Question 8:**

If either  $\vec{a} = \vec{0}$  or  $\vec{b} = \vec{0}$ , then  $\vec{a} \times \vec{b} = \vec{0}$ . Is the converse true? Justify your answer with an example.

Answer

Take any parallel non-zero vectors so that  $\vec{a} \times \vec{b} = \vec{0}$ .

$$\text{Let } \vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{b} = 4\hat{i} + 6\hat{j} + 8\hat{k}.$$

Then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 4 & 6 & 8 \end{vmatrix} = \hat{i}(24 - 24) - \hat{j}(16 - 16) + \hat{k}(12 - 12) = 0\hat{i} + 0\hat{j} + 0\hat{k} = \vec{0}$$

It can now be observed that:

$$|\vec{a}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$$

$$\therefore \vec{a} \neq \vec{0}$$

$$|\vec{b}| = \sqrt{4^2 + 6^2 + 8^2} = \sqrt{116}$$

$$\therefore \vec{b} \neq \vec{0}$$

Hence, the converse of the given statement need not be true.

**Question 9:**

Find the area of the triangle with vertices A (1, 1, 2), B (2, 3, 5) and C (1, 5, 5).

Answer

The vertices of triangle ABC are given as A (1, 1, 2), B (2, 3, 5), and C (1, 5, 5).

The adjacent sides  $\overline{AB}$  and  $\overline{BC}$  of  $\Delta ABC$  are given as:

$$\overline{AB} = (2-1)\hat{i} + (3-1)\hat{j} + (5-2)\hat{k} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\overline{BC} = (1-2)\hat{i} + (5-3)\hat{j} + (5-5)\hat{k} = -\hat{i} + 2\hat{j}$$

$$\text{Area of } \Delta ABC = \frac{1}{2} |\overline{AB} \times \overline{BC}|$$

$$\overline{AB} \times \overline{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 2 & 0 \end{vmatrix} = \hat{i}(-6) - \hat{j}(3) + \hat{k}(2+2) = -6\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\therefore |\overline{AB} \times \overline{BC}| = \sqrt{(-6)^2 + (-3)^2 + 4^2} = \sqrt{36+9+16} = \sqrt{61}$$

Hence, the area of  $\Delta ABC$  is  $\frac{\sqrt{61}}{2}$  square units.

**Question 10:**

Find the area of the parallelogram whose adjacent sides are determined by the vector

$$\vec{a} = \hat{i} - \hat{j} + 3\hat{k} \text{ and } \vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$$

Answer

The area of the parallelogram whose adjacent sides are  $\vec{a}$  and  $\vec{b}$  is  $|\vec{a} \times \vec{b}|$ .

Adjacent sides are given as:

$$\vec{a} = \hat{i} - \hat{j} + 3\hat{k} \text{ and } \vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = \hat{i}(-1+21) - \hat{j}(1-6) + \hat{k}(-7+2) = 20\hat{i} + 5\hat{j} - 5\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{20^2 + 5^2 + 5^2} = \sqrt{400 + 25 + 25} = 15\sqrt{2}$$

Hence, the area of the given parallelogram is  $15\sqrt{2}$  square units.

**Question 11:**

Let the vectors  $\vec{a}$  and  $\vec{b}$  be such that  $|\vec{a}| = 3$  and  $|\vec{b}| = \frac{\sqrt{2}}{3}$ , then  $\vec{a} \times \vec{b}$  is a unit vector, if

the angle between  $\vec{a}$  and  $\vec{b}$  is

(A)  $\frac{\pi}{6}$  (B)  $\frac{\pi}{4}$  (C)  $\frac{\pi}{3}$  (D)  $\frac{\pi}{2}$

Answer

It is given that  $|\vec{a}| = 3$  and  $|\vec{b}| = \frac{\sqrt{2}}{3}$ .

We know that  $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n}$ , where  $\hat{n}$  is a unit vector perpendicular to both  $\vec{a}$  and  $\vec{b}$  and  $\theta$  is the angle between  $\vec{a}$  and  $\vec{b}$ .

Now,  $\vec{a} \times \vec{b}$  is a unit vector if  $|\vec{a} \times \vec{b}| = 1$ .

$$|\vec{a} \times \vec{b}| = 1$$

$$\Rightarrow |\vec{a}||\vec{b}|\sin\theta = 1$$

$$\Rightarrow |\vec{a}||\vec{b}|\sin\theta = 1$$

$$\Rightarrow 3 \times \frac{\sqrt{2}}{3} \times \sin\theta = 1$$

$$\Rightarrow \sin\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Hence,  $\vec{a} \times \vec{b}$  is a unit vector if the angle between  $\vec{a}$  and  $\vec{b}$  is  $\frac{\pi}{4}$ .

The correct answer is B.

### Question 12:

Area of a rectangle having vertices A, B, C, and D with position vectors

$$-\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}, \hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}, \hat{i} - \frac{1}{2}\hat{j} + 4\hat{k} \text{ and } -\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k} \text{ respectively is}$$

(A)  $\frac{1}{2}$  (B) 1

(C) 2 (D) 4

Answer

The position vectors of vertices A, B, C, and D of rectangle ABCD are given as:

$$\overline{OA} = -\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}, \quad \overline{OB} = \hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}, \quad \overline{OC} = \hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}, \quad \overline{OD} = -\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$$

The adjacent sides  $\overline{AB}$  and  $\overline{BC}$  of the given rectangle are given as:

$$\overline{AB} = (1+1)\hat{i} + \left(\frac{1}{2} - \frac{1}{2}\right)\hat{j} + (4-4)\hat{k} = 2\hat{i}$$

$$\overline{BC} = (1-1)\hat{i} + \left(-\frac{1}{2} - \frac{1}{2}\right)\hat{j} + (4-4)\hat{k} = -\hat{j}$$

$$\therefore \overline{AB} \times \overline{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = \hat{k}(-2) = -2\hat{k}$$

$$|\overline{AB} \times \overline{BC}| = \sqrt{(-2)^2} = 2$$

Now, it is known that the area of a parallelogram whose adjacent sides are

$$\vec{a} \text{ and } \vec{b} \text{ is } |\vec{a} \times \vec{b}|.$$

Hence, the area of the given rectangle is  $|\overline{AB} \times \overline{BC}| = 2$  square units.

The correct answer is C.



## Miscellaneous Solutions

**Question 1:**

Write down a unit vector in XY-plane, making an angle of  $30^\circ$  with the positive direction of x-axis.

Answer

If  $\vec{r}$  is a unit vector in the XY-plane, then  $\vec{r} = \cos \theta \hat{i} + \sin \theta \hat{j}$ .

Here,  $\theta$  is the angle made by the unit vector with the positive direction of the x-axis.

Therefore, for  $\theta = 30^\circ$ :

$$\vec{r} = \cos 30^\circ \hat{i} + \sin 30^\circ \hat{j} = \frac{\sqrt{3}}{2} \hat{i} + \frac{1}{2} \hat{j}$$

Hence, the required unit vector is  $\frac{\sqrt{3}}{2} \hat{i} + \frac{1}{2} \hat{j}$

**Question 2:**

Find the scalar components and magnitude of the vector joining the points

$P(x_1, y_1, z_1)$  and  $Q(x_2, y_2, z_2)$ .

Answer

The vector joining the points  $P(x_1, y_1, z_1)$  and  $Q(x_2, y_2, z_2)$  can be obtained by,

$\overline{PQ}$  = Position vector of Q – Position vector of P

$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$|\overline{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Hence, the scalar components and the magnitude of the vector joining the given points

are respectively  $\{(x_2 - x_1), (y_2 - y_1), (z_2 - z_1)\}$  and  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ .

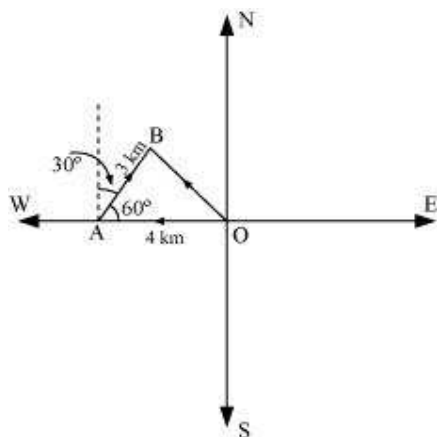
**Question 3:**

A girl walks 4 km towards west, then she walks 3 km in a direction  $30^\circ$  east of north and stops. Determine the girl's displacement from her initial point of departure.

Answer

Let O and B be the initial and final positions of the girl respectively.

Then, the girl's position can be shown as:



Now, we have:

$$\overline{OA} = -4\hat{i}$$

$$\overline{AB} = \hat{i}|\overline{AB}|\cos 60^\circ + \hat{j}|\overline{AB}|\sin 60^\circ$$

$$= \hat{i}3 \times \frac{1}{2} + \hat{j}3 \times \frac{\sqrt{3}}{2}$$

$$= \frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

By the triangle law of vector addition, we have:

$$\overline{OB} = \overline{OA} + \overline{AB}$$

$$= (-4\hat{i}) + \left(\frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}\right)$$

$$= \left(-4 + \frac{3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

$$= \left(\frac{-8+3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

$$= \frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

Hence, the girl's displacement from her initial point of departure is

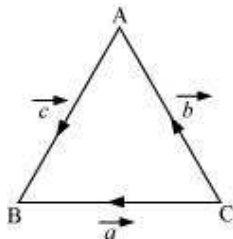
$$\frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

**Question 4:**

If  $\vec{a} = \vec{b} + \vec{c}$ , then is it true that  $|\vec{a}| = |\vec{b}| + |\vec{c}|$ ? Justify your answer.

Answer

In  $\triangle ABC$ , let  $\overrightarrow{CB} = \vec{a}$ ,  $\overrightarrow{CA} = \vec{b}$ , and  $\overrightarrow{AB} = \vec{c}$  (as shown in the following figure).



Now, by the triangle law of vector addition, we have  $\vec{a} = \vec{b} + \vec{c}$ .

It is clearly known that  $|\vec{a}|$ ,  $|\vec{b}|$ , and  $|\vec{c}|$  represent the sides of  $\triangle ABC$ .

Also, it is known that the sum of the lengths of any two sides of a triangle is greater than the third side.

$$\therefore |\vec{a}| < |\vec{b}| + |\vec{c}|$$

Hence, it is not true that  $|\vec{a}| = |\vec{b}| + |\vec{c}|$ .

**Question 5:**

Find the value of  $x$  for which  $x(\hat{i} + \hat{j} + \hat{k})$  is a unit vector.

Answer

$$x(\hat{i} + \hat{j} + \hat{k}) \text{ is a unit vector if } |x(\hat{i} + \hat{j} + \hat{k})| = 1$$

Now,

$$\begin{aligned} |x(\hat{i} + \hat{j} + \hat{k})| &= 1 \\ \Rightarrow \sqrt{x^2 + x^2 + x^2} &= 1 \\ \Rightarrow \sqrt{3x^2} &= 1 \\ \Rightarrow \sqrt{3}x &= 1 \\ \Rightarrow x &= \pm \frac{1}{\sqrt{3}} \end{aligned}$$

Hence, the required value of  $x$  is  $\pm \frac{1}{\sqrt{3}}$ .

### Question 6:

Find a vector of magnitude 5 units, and parallel to the resultant of the vectors

$$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k} \text{ and } \vec{b} = \hat{i} - 2\hat{j} + \hat{k}.$$

Answer

We have,

$$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k} \text{ and } \vec{b} = \hat{i} - 2\hat{j} + \hat{k}$$

Let  $\vec{c}$  be the resultant of  $\vec{a}$  and  $\vec{b}$ .

Then,

$$\vec{c} = \vec{a} + \vec{b} = (2+1)\hat{i} + (3-2)\hat{j} + (-1+1)\hat{k} = 3\hat{i} + \hat{j}$$

$$\therefore |\vec{c}| = \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$$

$$\therefore \hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{(3\hat{i} + \hat{j})}{\sqrt{10}}$$

Hence, the vector of magnitude 5 units and parallel to the resultant of vectors  $\vec{a}$  and  $\vec{b}$  is

$$\pm 5 \cdot \hat{c} = \pm 5 \cdot \frac{1}{\sqrt{10}}(3\hat{i} + \hat{j}) = \pm \frac{3\sqrt{10}\hat{i}}{2} \pm \frac{\sqrt{10}\hat{j}}{2}.$$

**Question 7:**

If  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ ,  $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$  and  $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$ , find a unit vector parallel to the vector  $2\vec{a} - \vec{b} + 3\vec{c}$ .

Answer

We have,

$$\begin{aligned}\vec{a} &= \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} + 3\hat{k} \text{ and } \vec{c} = \hat{i} - 2\hat{j} + \hat{k} \\ 2\vec{a} - \vec{b} + 3\vec{c} &= 2(\hat{i} + \hat{j} + \hat{k}) - (2\hat{i} - \hat{j} + 3\hat{k}) + 3(\hat{i} - 2\hat{j} + \hat{k}) \\ &= 2\hat{i} + 2\hat{j} + 2\hat{k} - 2\hat{i} + \hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k} \\ &= 3\hat{i} - 3\hat{j} + 2\hat{k}\end{aligned}$$

$$|2\vec{a} - \vec{b} + 3\vec{c}| = \sqrt{3^2 + (-3)^2 + 2^2} = \sqrt{9+9+4} = \sqrt{22}$$

Hence, the unit vector along  $2\vec{a} - \vec{b} + 3\vec{c}$  is

$$\frac{2\vec{a} - \vec{b} + 3\vec{c}}{|2\vec{a} - \vec{b} + 3\vec{c}|} = \frac{3\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{22}} = \frac{3}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{2}{\sqrt{22}}\hat{k}.$$

**Question 8:**

Show that the points A (1, -2, -8), B (5, 0, -2) and C (11, 3, 7) are collinear, and find the ratio in which B divides AC.

Answer

The given points are A (1, -2, -8), B (5, 0, -2), and C (11, 3, 7).

$$\therefore \vec{AB} = (5-1)\hat{i} + (0+2)\hat{j} + (-2+8)\hat{k} = 4\hat{i} + 2\hat{j} + 6\hat{k}$$

$$\vec{BC} = (11-5)\hat{i} + (3-0)\hat{j} + (7+2)\hat{k} = 6\hat{i} + 3\hat{j} + 9\hat{k}$$

$$\vec{AC} = (11-1)\hat{i} + (3+2)\hat{j} + (7+8)\hat{k} = 10\hat{i} + 5\hat{j} + 15\hat{k}$$

$$|\vec{AB}| = \sqrt{4^2 + 2^2 + 6^2} = \sqrt{16+4+36} = \sqrt{56} = 2\sqrt{14}$$

$$|\vec{BC}| = \sqrt{6^2 + 3^2 + 9^2} = \sqrt{36+9+81} = \sqrt{126} = 3\sqrt{14}$$

$$|\vec{AC}| = \sqrt{10^2 + 5^2 + 15^2} = \sqrt{100+25+225} = \sqrt{350} = 5\sqrt{14}$$

$$\therefore |\vec{AC}| = |\vec{AB}| + |\vec{BC}|$$

Thus, the given points A, B, and C are collinear.

Now, let point B divide AC in the ratio  $\lambda:1$ . Then, we have:

$$\overline{OB} = \frac{\lambda\overline{OC} + \overline{OA}}{(\lambda+1)}$$

$$\Rightarrow 5\hat{i} - 2\hat{k} = \frac{\lambda(11\hat{i} + 3\hat{j} + 7\hat{k}) + (\hat{i} - 2\hat{j} - 8\hat{k})}{\lambda+1}$$

$$\Rightarrow (\lambda+1)(5\hat{i} - 2\hat{k}) = 11\lambda\hat{i} + 3\lambda\hat{j} + 7\lambda\hat{k} + \hat{i} - 2\hat{j} - 8\hat{k}$$

$$\Rightarrow 5(\lambda+1)\hat{i} - 2(\lambda+1)\hat{k} = (11\lambda+1)\hat{i} + (3\lambda-2)\hat{j} + (7\lambda-8)\hat{k}$$

On equating the corresponding components, we get:

$$5(\lambda+1) = 11\lambda+1$$

$$\Rightarrow 5\lambda+5 = 11\lambda+1$$

$$\Rightarrow 6\lambda = 4$$

$$\Rightarrow \lambda = \frac{4}{6} = \frac{2}{3}$$

Hence, point B divides AC in the ratio 2:3.

### Question 9:

Find the position vector of a point R which divides the line joining two points P and Q

whose position vectors are  $(2\vec{a} + \vec{b})$  and  $(\vec{a} - 3\vec{b})$  externally in the ratio 1: 2. Also, show that P is the mid point of the line segment RQ.

Answer

It is given that  $\overline{OP} = 2\vec{a} + \vec{b}$ ,  $\overline{OQ} = \vec{a} - 3\vec{b}$ .

It is given that point R divides a line segment joining two points P and Q externally in the ratio 1: 2. Then, on using the section formula, we get:

$$\overline{OR} = \frac{2(2\vec{a} + \vec{b}) - (\vec{a} - 3\vec{b})}{2-1} = \frac{4\vec{a} + 2\vec{b} - \vec{a} + 3\vec{b}}{1} = 3\vec{a} + 5\vec{b}$$

Therefore, the position vector of point R is  $3\vec{a} + 5\vec{b}$ .

$$\text{Position vector of the mid-point of RQ} = \frac{\overline{OQ} + \overline{OR}}{2}$$

$$\begin{aligned}
 &= \frac{(\vec{a} - 3\vec{b}) + (3\vec{a} + 5\vec{b})}{2} \\
 &= 2\vec{a} + \vec{b} \\
 &= \overline{OP}
 \end{aligned}$$

Hence, P is the mid-point of the line segment RQ.

### Question 10:

The two adjacent sides of a parallelogram are  $2\hat{i} - 4\hat{j} + 5\hat{k}$  and  $\hat{i} - 2\hat{j} - 3\hat{k}$ .

Find the unit vector parallel to its diagonal. Also, find its area.

Answer

Adjacent sides of a parallelogram are given as:  $\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$  and  $\vec{b} = \hat{i} - 2\hat{j} - 3\hat{k}$

Then, the diagonal of a parallelogram is given by  $\vec{a} + \vec{b}$ .

$$\vec{a} + \vec{b} = (2+1)\hat{i} + (-4-2)\hat{j} + (5-3)\hat{k} = 3\hat{i} - 6\hat{j} + 2\hat{k}$$

Thus, the unit vector parallel to the diagonal is

$$\frac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|} = \frac{3\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{3^2 + (-6)^2 + 2^2}} = \frac{3\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{9 + 36 + 4}} = \frac{3\hat{i} - 6\hat{j} + 2\hat{k}}{7} = \frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}.$$

$\therefore$  Area of parallelogram ABCD =  $|\vec{a} \times \vec{b}|$

$$\begin{aligned}
 \vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 5 \\ 1 & -2 & -3 \end{vmatrix} \\
 &= \hat{i}(12+10) - \hat{j}(-6-5) + \hat{k}(-4+4) \\
 &= 22\hat{i} + 11\hat{j} \\
 &= 11(2\hat{i} + \hat{j})
 \end{aligned}$$

$$\therefore |\vec{a} \times \vec{b}| = 11\sqrt{2^2 + 1^2} = 11\sqrt{5}$$

Hence, the area of the parallelogram is  $11\sqrt{5}$  square units.

**Question 11:**

Show that the direction cosines of a vector equally inclined to the axes OX, OY and OZ

are  $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Answer

Let a vector be equally inclined to axes OX, OY, and OZ at angle  $\alpha$ .

Then, the direction cosines of the vector are  $\cos \alpha$ ,  $\cos \alpha$ , and  $\cos \alpha$ .

Now,

$$\cos^2 \alpha + \cos^2 \alpha + \cos^2 \alpha = 1$$

$$\Rightarrow 3 \cos^2 \alpha = 1$$

$$\Rightarrow \cos \alpha = \frac{1}{\sqrt{3}}$$

Hence, the direction cosines of the vector which are equally inclined to the axes

are  $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ .

**Question 12:**

Let  $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$ ,  $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$  and  $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ . Find a vector  $\vec{d}$  which is perpendicular to both  $\vec{a}$  and  $\vec{b}$ , and  $\vec{c} \cdot \vec{d} = 15$ .

Answer

Let  $\vec{d} = d_1\hat{i} + d_2\hat{j} + d_3\hat{k}$ .

Since  $\vec{d}$  is perpendicular to both  $\vec{a}$  and  $\vec{b}$ , we have:

$$\vec{d} \cdot \vec{a} = 0$$

$$\Rightarrow d_1 + 4d_2 + 2d_3 = 0 \quad \dots(i)$$

And,

$$\vec{d} \cdot \vec{b} = 0$$

$$\Rightarrow 3d_1 - 2d_2 + 7d_3 = 0 \quad \dots(ii)$$

Also, it is given that:

$$\vec{c} \cdot \vec{d} = 15$$

$$\Rightarrow 2d_1 - d_2 + 4d_3 = 15 \quad \dots(iii)$$



On solving (i), (ii), and (iii), we get:

$$d_1 = \frac{160}{3}, d_2 = -\frac{5}{3} \text{ and } d_3 = -\frac{70}{3}$$

$$\therefore \vec{d} = \frac{160}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{70}{3}\hat{k} = \frac{1}{3}(160\hat{i} - 5\hat{j} - 70\hat{k})$$

Hence, the required vector is  $\frac{1}{3}(160\hat{i} - 5\hat{j} - 70\hat{k})$ .

**Question 13:**

The scalar product of the vector  $\hat{i} + \hat{j} + \hat{k}$  with a unit vector along the sum of vectors

$2\hat{i} + 4\hat{j} - 5\hat{k}$  and  $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$  is equal to one. Find the value of  $\lambda$ .

Answer

$$(2\hat{i} + 4\hat{j} - 5\hat{k}) + (\lambda\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= (2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}$$

Therefore, unit vector along  $(2\hat{i} + 4\hat{j} - 5\hat{k}) + (\lambda\hat{i} + 2\hat{j} + 3\hat{k})$  is given as:

$$\frac{(2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2 + \lambda)^2 + 6^2 + (-2)^2}} = \frac{(2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{4 + 4\lambda + \lambda^2 + 36 + 4}} = \frac{(2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{\lambda^2 + 4\lambda + 44}}$$

Scalar product of  $(\hat{i} + \hat{j} + \hat{k})$  with this unit vector is 1.

$$\Rightarrow (\hat{i} + \hat{j} + \hat{k}) \cdot \frac{(2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{\lambda^2 + 4\lambda + 44}} = 1$$

$$\Rightarrow \frac{(2 + \lambda) + 6 - 2}{\sqrt{\lambda^2 + 4\lambda + 44}} = 1$$

$$\Rightarrow \sqrt{\lambda^2 + 4\lambda + 44} = \lambda + 6$$

$$\Rightarrow \lambda^2 + 4\lambda + 44 = (\lambda + 6)^2$$

$$\Rightarrow \lambda^2 + 4\lambda + 44 = \lambda^2 + 12\lambda + 36$$

$$\Rightarrow 8\lambda = 8$$

$$\Rightarrow \lambda = 1$$

Hence, the value of  $\lambda$  is 1.

**Question 14:**

If  $\vec{a}, \vec{b}, \vec{c}$  are mutually perpendicular vectors of equal magnitudes, show that the vector  $\vec{a} + \vec{b} + \vec{c}$  is equally inclined to  $\vec{a}, \vec{b}$  and  $\vec{c}$ .

Answer

Since  $\vec{a}, \vec{b}$ , and  $\vec{c}$  are mutually perpendicular vectors, we have

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0.$$

It is given that:

$$|\vec{a}| = |\vec{b}| = |\vec{c}|$$

Let vector  $\vec{a} + \vec{b} + \vec{c}$  be inclined to  $\vec{a}, \vec{b}$ , and  $\vec{c}$  at angles  $\theta_1, \theta_2$ , and  $\theta_3$  respectively.

Then, we have:

$$\begin{aligned} \cos \theta_1 &= \frac{(\vec{a} + \vec{b} + \vec{c}) \cdot \vec{a}}{|\vec{a} + \vec{b} + \vec{c}| |\vec{a}|} = \frac{\vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{a} + \vec{c} \cdot \vec{a}}{|\vec{a} + \vec{b} + \vec{c}| |\vec{a}|} \\ &= \frac{|\vec{a}|^2}{|\vec{a} + \vec{b} + \vec{c}| |\vec{a}|} \quad [\vec{b} \cdot \vec{a} = \vec{c} \cdot \vec{a} = 0] \end{aligned}$$

$$\begin{aligned} \cos \theta_2 &= \frac{(\vec{a} + \vec{b} + \vec{c}) \cdot \vec{b}}{|\vec{a} + \vec{b} + \vec{c}| |\vec{b}|} = \frac{\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{b}}{|\vec{a} + \vec{b} + \vec{c}| |\vec{b}|} \\ &= \frac{|\vec{b}|^2}{|\vec{a} + \vec{b} + \vec{c}| |\vec{b}|} \quad [\vec{a} \cdot \vec{b} = \vec{c} \cdot \vec{b} = 0] \end{aligned}$$

$$\begin{aligned} \cos \theta_3 &= \frac{(\vec{a} + \vec{b} + \vec{c}) \cdot \vec{c}}{|\vec{a} + \vec{b} + \vec{c}| |\vec{c}|} = \frac{\vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{c}}{|\vec{a} + \vec{b} + \vec{c}| |\vec{c}|} \\ &= \frac{|\vec{c}|^2}{|\vec{a} + \vec{b} + \vec{c}| |\vec{c}|} \quad [\vec{a} \cdot \vec{c} = \vec{b} \cdot \vec{c} = 0] \\ &= \frac{|\vec{c}|}{|\vec{a} + \vec{b} + \vec{c}|} \end{aligned}$$

Now, as  $|\vec{a}| = |\vec{b}| = |\vec{c}|$ ,  $\cos \theta_1 = \cos \theta_2 = \cos \theta_3$ .  
 $\therefore \theta_1 = \theta_2 = \theta_3$

Hence, the vector  $(\vec{a} + \vec{b} + \vec{c})$  is equally inclined to  $\vec{a}, \vec{b}$ , and  $\vec{c}$ .

**Question 15:**

Prove that  $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$ , if and only if  $\vec{a}, \vec{b}$  are perpendicular, given  $\vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}$ .

Answer

$$\begin{aligned} (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) &= |\vec{a}|^2 + |\vec{b}|^2 \\ \Leftrightarrow \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} &= |\vec{a}|^2 + |\vec{b}|^2 && \text{[Distributivity of scalar products over addition]} \\ \Leftrightarrow |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 &= |\vec{a}|^2 + |\vec{b}|^2 && \text{[}\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} \text{ (Scalar product is commutative)]} \\ \Leftrightarrow 2\vec{a} \cdot \vec{b} &= 0 \\ \Leftrightarrow \vec{a} \cdot \vec{b} &= 0 \\ \therefore \vec{a} \text{ and } \vec{b} &\text{ are perpendicular.} && \text{[}\vec{a} \neq \vec{0}, \vec{b} \neq \vec{0} \text{ (Given)]} \end{aligned}$$

**Question 16:**

If  $\theta$  is the angle between two vectors  $\vec{a}$  and  $\vec{b}$ , then  $\vec{a} \cdot \vec{b} \geq 0$  only when

(A)  $0 < \theta < \frac{\pi}{2}$  (B)  $0 \leq \theta \leq \frac{\pi}{2}$

(C)  $0 < \theta < \pi$  (D)  $0 \leq \theta \leq \pi$

Answer

Let  $\theta$  be the angle between two vectors  $\vec{a}$  and  $\vec{b}$ .

Then, without loss of generality,  $\vec{a}$  and  $\vec{b}$  are non-zero vectors so

that  $|\vec{a}|$  and  $|\vec{b}|$  are positive.

It is known that  $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$ .

$$\therefore \vec{a} \cdot \vec{b} \geq 0$$

$$\Rightarrow |\vec{a}||\vec{b}|\cos\theta \geq 0$$

$$\Rightarrow \cos\theta \geq 0 \quad [|\vec{a}| \text{ and } |\vec{b}| \text{ are positive}]$$

$$\Rightarrow 0 \leq \theta \leq \frac{\pi}{2}$$

Hence,  $\vec{a} \cdot \vec{b} \geq 0$  when  $0 \leq \theta \leq \frac{\pi}{2}$ .

The correct answer is B.

**Question 17:**

Let  $\vec{a}$  and  $\vec{b}$  be two unit vectors and  $\theta$  is the angle between them. Then  $\vec{a} + \vec{b}$  is a unit vector if

(A)  $\theta = \frac{\pi}{4}$  (B)  $\theta = \frac{\pi}{3}$  (C)  $\theta = \frac{\pi}{2}$  (D)  $\theta = \frac{2\pi}{3}$

Answer

Let  $\vec{a}$  and  $\vec{b}$  be two unit vectors and  $\theta$  be the angle between them.

Then,  $|\vec{a}| = |\vec{b}| = 1$ .

Now,  $\vec{a} + \vec{b}$  is a unit vector if  $|\vec{a} + \vec{b}| = 1$ .

$$\begin{aligned}
 |\vec{a} + \vec{b}| &= 1 \\
 \Rightarrow (\vec{a} + \vec{b})^2 &= 1 \\
 \Rightarrow (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) &= 1 \\
 \Rightarrow \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} &= 1 \\
 \Rightarrow |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 &= 1 \\
 \Rightarrow 1^2 + 2|\vec{a}||\vec{b}|\cos\theta + 1^2 &= 1 \\
 \Rightarrow 1 + 2 \cdot 1 \cdot 1 \cos\theta + 1 &= 1 \\
 \Rightarrow \cos\theta &= -\frac{1}{2} \\
 \Rightarrow \theta &= \frac{2\pi}{3}
 \end{aligned}$$

Hence,  $\vec{a} + \vec{b}$  is a unit vector if  $\theta = \frac{2\pi}{3}$ .

The correct answer is D.

#### Question 18:

The value of  $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$  is

(A) 0 (B) -1 (C) 1 (D) 3

Answer

$$\begin{aligned}
 &\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j}) \\
 &= \hat{i} \cdot \hat{i} + \hat{j} \cdot (-\hat{j}) + \hat{k} \cdot \hat{k} \\
 &= 1 - \hat{j} \cdot \hat{j} + 1 \\
 &= 1 - 1 + 1 \\
 &= 1
 \end{aligned}$$

The correct answer is C.

#### Question 19:

If  $\theta$  is the angle between any two vectors  $\vec{a}$  and  $\vec{b}$ , then  $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$  when  $\theta$  is equal to

(A) 0 (B)  $\frac{\pi}{4}$  (C)  $\frac{\pi}{2}$  (D)  $\pi$

Answer

Let  $\theta$  be the angle between two vectors  $\vec{a}$  and  $\vec{b}$ .

Then, without loss of generality,  $\vec{a}$  and  $\vec{b}$  are non-zero vectors, so that  $|\vec{a}|$  and  $|\vec{b}|$  are positive

$$|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$$

$$\Rightarrow |\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\sin\theta$$

$$\Rightarrow \cos\theta = \sin\theta \quad [|\vec{a}| \text{ and } |\vec{b}| \text{ are positive}]$$

$$\Rightarrow \tan\theta = 1$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Hence,  $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$  when  $\theta$  is equal to  $\frac{\pi}{4}$ .

The correct answer is B.